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A Strategic Prioritization Approach to Airline Scheduling During Disruptions

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A Strategic Prioritization Approach to Airline Scheduling During Disruptions

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THESIS

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

MASTER OF SCIENCE IN ENGINEERING

THE UNIVERSITY OF TEXAS AT AUSTIN

May 2015

Dedicated to my family

Acknowledgments

I would like to take this opportunity to thank my thesis advisors, Professor Douglas Fearing and Professor Ned Dimitrov for providing me the opportunity to do research work under their supervision. Working with Professor Fearing has truly been a memorable as well as an enjoyable learning experience for me. Over the course of my thesis, I have learnt a great deal from him not only about research, but also about the airline industry. I am extremely grateful for the incredible support, guidance and motivation which he has provided over the last one year. I am also very thankful to Professor Dimitrov, whose guidance and suggestions were invaluable for my thesis work. I am especially thankful to Dr. Ian Kash, whose valuable insights and expertise in the subject matter were instrumental for me in proving the game theoretic results for my thesis.

My last two years were one of the best experiences of my life. And they would not have been the same had it not been for my friends at UT. I am very thankful to each and everyone of them for their help and encouragement. Finally, I would like to sincerely thank my family for their unconditional love and support, and for always encouraging me to follow my dreams.

A Strategic Prioritization Approach to Airline Scheduling During Disruptions

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The University of Texas at Austin, 2015

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Air disruption scenarios due to inclement weather or air traffic congestion can result in significant imbalances in the demands and capacities of the affected airports. The Federal Aviation Administration (FAA) resolves such imbalances by implementing the Ground Delay Programs (GDP). In a GDP, the FAA first assigns new arrival slots to the airlines using the Ration By Schedule (RBS) approach, which is an allocation procedure that assigns slots to incoming flights based on a First-Scheduled, First-Served (FSFS) criterion. The FAA uses these new arrival slots to determine the expected delays and accommodates them as ground delays at departure airports.

The notion of FSFS that forms the basis of RBS, is considered to be an industry standard of fairness. One of the major shortcomings of the RBS approach is that it does not distinguish flights based on factors like aircraft size, number of passengers, future aircraft schedules, etc. This results in an inefficient utilization of the airport capacities. To address this concern, Fearing and Kash [1] proposed a two-stage, non-monetary strategic prioritization game in which airlines could participate and bid

for priorities at different airports by taking into account their internal costs. This approach has several advantages over different market-based mechanisms like slot auctions, congestion pricing and slot exchanges.

In this thesis, therefore, we develop their approach further both mathematically as well as empirically. Specifically, we prove that a pure strategy Nash equilibrium exists in the second stage of the game for the general multiple airlines and multiple airports case. In addition, by imposing the diagonal strict concavity conditions on the airlines' payoffs, we show that this pure strategy Nash equilibrium is unique for the two-airlines case. Our experimental simulations on historical data further show that this approach can achieve significant congestion cost benefits in comparison to the current RBS procedure.

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Chapter 1

Introduction

Over the last few decades, the airline industry in the United States has grown tremendously. To provide a perspective on the current scale of operations, in 2014, around 80 different commercial air carriers in the US carried over 760 million passengers to nearly 600 different locations within the US, averaging about 30,000 flights per day (Bureau of Transportation Statistics (BTS) [2]). According to the Federal Aviation Administration (FAA) [3], the US civil aviation industry in 2012 contributed \$1.5 trillion to the total economic activity and accounted for 5.4% of the US Gross Domestic Product. Thus, it is clear that the civil aviation in the US, in addition to providing a means of transportation to thousands of travellers every day, also contributes significantly to the growth of the economy by creating employment opportunities and supporting several other related industries like manufacturing and tourism. Therefore, a smooth and efficient functioning of the industry is of utmost importance.

To keep up with the steady growth in the number of air travellers (refer to Figure 1.1) over the last decade, more and more flights have been scheduled, because of which air traffic congestion has emerged as one of the key concerning issues for the regulators to deal with. According to the Bureau of Transportation Statistics (BTS)

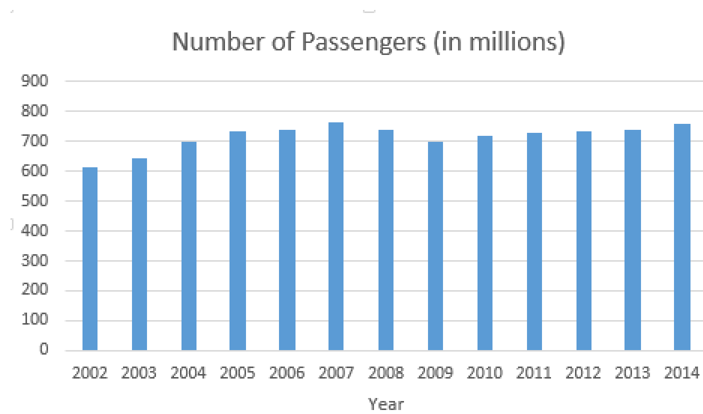

Figure 1.1: Yearly trend of the total number of passengers carried by US Airlines DATA SOURCE: Bureau of Transportation Statistics (BTS) [2]

[2], in 2013, nearly 20 percent of the scheduled flights were delayed by more than 15 minutes and an additional 1.5% were cancelled. Ball et al. [4] estimated the total direct costs associated with these flight delays and cancellations for the year 2007 to be around $30 billion, out of which the approximate costs to the airlines and passengers were $8.3 billion and $17 billion respectively. To put these numbers into context, consider Figure 1.2 which shows the total operating profits for the airlines in the last 15 years. It can be clearly seen from the figure that the overall profits for the airline industry are in about the same range as the costs incurred due to delays. In addition, the fluctuating pattern of the profits reflects the turmoil that the aviation industry has been going through, particularly due to external factors like the September 11, 2001 terrorist attacks and the Financial Crisis of 2008, along with a host of other factors like rising fuel prices, increasing competition and infrastructure limitations. Thus, taking into view the uncertain environment in which the airline operate in, it becomes even more imperative to optimize the existing airline operations as much as

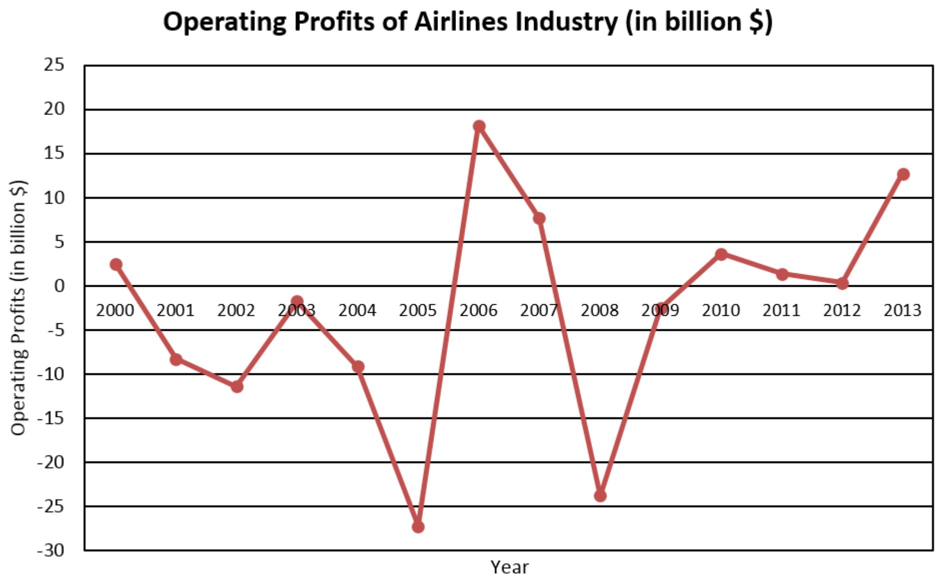

Figure 1.2: Airline Industry Profits from 2000-2013.
DATA SOURCE: Bureau of Transportation Statistics (BTS) [2]

possible.

The issue of air traffic congestion in the National Air Transportation System (NATS) is mainly the outcome of a disproportionately large increase in the number of scheduled flights, in comparison to the existing capacities of the air transportation resources. Over the years, as the number of air travellers has grown, more and more flights have been scheduled to meet the increasing demand. In contrast, however, the capacities of the air transportation resources have remained fairly stagnant. Due to this reason, many of the major airports in the US today operate at full or nearly full capacities even during normal operations. This inevitably leads to delays during peak hours or during severe weather conditions when the capacities of the resources are considerably reduced. Figure 1.3 shows the major causes (along with their complete

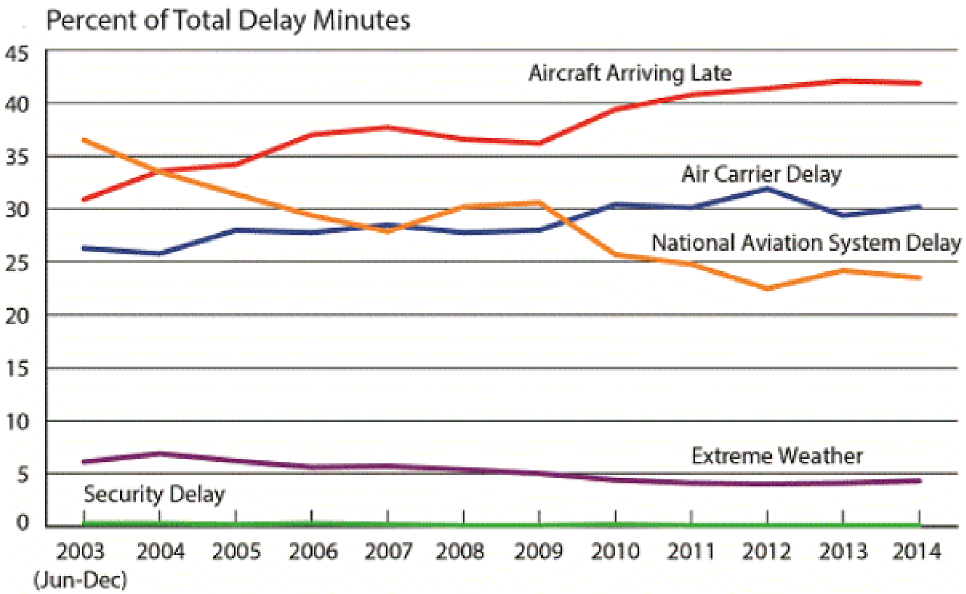

Figure 1.3: Causes of Airline Delay.
Air Carrier delays include circumstances within the airline's control (e.g. maintenance or crew problems, aircraft cleaning, baggage loading, fueling, etc.). Extreme Weather delays are due to significant meteorological conditions that causes delays like tornado, blizzard or hurricane. National Aviation System delays include a broad set of conditions, such as non-extreme weather conditions, airport operations, heavy traffic volume, etc. Late-arriving Aircraft delays occur when a previous flight with same aircraft arrived late, causing the present flight to depart late. Security delays are caused by evacuation of a terminal or concourse, re-boarding of aircraft because of security breach.
FIGURE AND CAPTION SOURCE: Bureau of Transportation Statistics (BTS) [2]

descriptions in the caption) for flights delays in the past decade, as noted by the FAA. A key observation that can be made from the figure is that only the Air Carrier delays, constituting about 30% of the total flight delays, are under the control of the airlines. The remaining 70% of the delays are disruptions caused either due to bad weather or air traffic congestion or security issues, all of which are outside the control of the airlines. Another important observation that we can make from the figure is that amongst all the factors, the Late Arrival of Aircraft contributes the most to flight delays. Even though the FAA does not receive data from the airlines regarding the causes which lead to an aircraft's late arrival, this factor is particularly important and relevant as it clearly indicates the adverse effects of delay propagation in the airports network. We discuss this in greater detail in Chapter 4, where we use this observation to define one of our simulation strategies for prioritizing flights.

Based on the time scale of implementation, Odoni [5] proposed long term, medium-term and short-term approaches for the mitigation of air traffic congestion. The long-term approaches, which have a time span of around 5-10 years, involve expanding the capacity of existing system infrastructure through the development of new airports and runways or by using more advanced Air Traffic Control (ATC) technologies that help in optimizing the usage of existing resources. The medium term approaches (with a time span between 6 months and 2 years) are aimed at creating more uniform traffic demand patterns during the day at airports by reducing the congestion during peak hours through slot auctioning or congestion pricing mechanisms. The short term approaches (time scale of 6-12 hours) involve optimizing the operations by making changes in the existing Air Traffic Management programs.

There are several issues, which make the implementation of long-term approaches difficult in practice. Firstly, the development of new airports and runways are high-budget, long-term projects, which require substantial funds and take several years to complete. In addition, the busiest airports are usually also located in the most congested parts of cities where space constraints are severe, thus making expansion an infeasible alternative. In this thesis, therefore, we mainly focus on short-term and medium-term approaches to alleviate air traffic congestion.

1.1 Air Traffic Management (ATM)

In the United States, the Federal Aviation Administration (FAA) is the main regulatory body, which coordinates all activities pertaining to civil aviation. In particular, the main objective of the FAA is to create an aerospace environment which facilitates safe and efficient movement of air traffic . The FAA achieves this objective by implementing a set of practices, which collectively form the Air Traffic Management (ATM) program. The ATM practices can be broadly classified into types: (1) Air Traffic Control (ATC) procedures and (2) Air Traffic Flow Management (ATFM) programs. The ATC procedures are tactical procedures, which track real-time movements of operating aircrafts and ensure that safe separation exists between airborne aircrafts at any given time. The Air Traffic Flow Management programs, on the other hand, are strategic practices which streamline the air traffic by resolving imbalances in the demands and capacities of air transportation resources, namely the airports and air sectors. From a standpoint of improving the overall system efficiency, the ATFM programs, thus offer a much larger scope as compared to the ATC procedures.

1.1.1 Traffic Flow Management (TFM)

As described above, the FAA implements ATFM programs on the day of operations to control air traffic congestion at airports and air sectors. These programs are aimed at improving air safety while at the same time minimizing costs associated with flight delays. Under the TFM framework, the FAA determines the maximum arrival capacity for all the resources within the National Airspace System (NAS) taking into account a number of a factors like weather conditions, runway configurations, number of runways and scheduled air staff (Gentry et al. [6], Barnhart et al. [7]). In addition, the FAA also keeps track of the current as well as projected demands for these resources and whenever the expected demand increases significantly with respect to the projected capacity, it implements the ATFM program. A key thing to note over here is that the ATFM programs are implemented only when there are significantly large imbalances between resource demand and capacity; for smaller imbalances, the FAA relies on ATC procedures to manage the air traffic.

There are two main initiatives, which have been developed by the FAA under the ATFM program and are currently in practice. The first initiative, known as the Ground Delay Program (GDP), was brought into operation in 1981 after a strike by the Air Traffic Controllers union. Under this initiative, the FAA controls those airports that have more projected demands than the capacities. Usually, such a situation arises due to inclement weather conditions or due to some other reasons like closed runways, etc. The FAA computes updated (reduced) arrival capacities for such airports and accordingly, assigns new (delayed) arrival slots to the incoming flights at that airport. Based on the delays associated with each flight, it then holds the delayed flights at their departure airports. This practice helps in translating the airborne delays to ground delays, thereby improving air safety and reducing fuel costs. The second initiative, known as the Airspace Flow Program (AFP), is very similar to a GDP. However, in an AFP, the FAA controls a Flow Constrained Area (FCA), which is a reduced capacity air sector within the NAS instead of an airport.

1.1.2 Collaborative Decision Making (CDM)

Until the mid 1990s, the decisions involving the control and management of air traffic congestion were made centrally by the FAA. However, at that time, due to the nonexistence of a real-time information sharing platform, the FAA did not have updated information about the current schedules of the airlines and had to rely on airline schedules published months in advance in the Official Airline Guide (Chang et al. [8]). Similarly, the airlines also had no knowledge about the resource demands (which the FAA had) and had to rely on weather forecasts to estimate the utilization of the resources. However, with the introduction of the Collaborative Decision Making (CDM) paradigm, several new procedures came into practice, which emphasized an improved real-time data exchange system between the FAA and airlines, and also called for a greater involvement of airlines on matters that had a direct economic impact on them. A key enhancement that CDM brought in the GDP context was the way in which arrival slots were allocated to the airlines. Before the introduction of CDM, the arrival slots were allocated to *individual flights* based on their most recent estimated time of arrival. This disincentivized the airlines to provide updated information regarding cancelled and delayed flights, as it meant that the airlines would lose their original (earlier scheduled) allocated arrival slots. Under the CDM approach, two modifications were made in this allocation procedure: firstly, the arrival slots were now considered to be allocated to the *airlines* (rather than *individual flights*) and secondly, the slots allocation procedure was now based on the *original Scheduled Time of Arrival* of the flights instead of their most *recent estimated arrival time*.

The GDP enhancements under the CDM approach, thus, paved the path for a three-stage procedure for slots allocation. In the first stage of this procedure, the FAA allocates new arrival slots according to the projected reduced arrival capacities for the controlled airport. The allocation of new arrival slots to airlines is done in the same order as the order of flights in the original flight schedule and is therefore, termed as Ration By Schedule (RBS). This allotment procedure, based on the notion of first-scheduled, first-served, is considered to be the accepted standard for fairness in the airline industry as it preserves the initial ordering of the scheduled flights.

We now consider a small example to illustrate the implementation of RBS procedure at a controlled airport with four incoming scheduled flights. In this example, we assume that due to bad weather, the arrival capacity at this airport has been has been reduced to 1 flight per 10 minutes. The FAA, therefore, implements RBS on the original Scheduled Times of Arrival (STA) shown in column 1 of Table 1.1 and assigns new Controlled Times of Arrival (CTAs) as given in column 2 of Table 1.1.

As described above, the RBS allocation is interpreted as an allocation of slots to airlines rather than an allocation of slots to individual flights. Thus, in the second stage, the airlines have the opportunity to reschedule (substitute or cancel) their

Flight	**Original Schedule**	**RBS**
A1	7:00 PM	7:00 PM
A2	7:06 PM	7:10 PM
B1	7:10 PM	7:20 PM
B2	7:12 PM	7:30 PM

Table 1.1: Original and RBS schedules for a controlled airport with reduced capacity of 1 flight per 10 min

flights within the slots initially allocated to them in the first stage. The only restriction while swapping flight slots is that each flight must be scheduled at or after its original arrival time in the new schedule. Thus, in our example, airline B can reschedule its flights B1 and B2 in 7:30 PM and 7:20 PM slots respectively. However, airline A cannot do the same as swapping slots between A1 and A2 results in A2 being scheduled at 7:00 PM, which is earlier than its original scheduled arrival time of 7:06 PM.

Due to flight cancellations or delays, the resulting schedule at the end of the second stage may contain gaps or vacant slots. Thus, in such cases, the FAA optimizes the resulting schedules in the third stage by iteratively moving the flights up in the schedule to fill the vacant slots. This procedure is known as compression and is described in detail in Vossen and Ball [9]. After the compression stage, the airlines are notified about the updated schedule and the final two stages in the procedure are repeated to allow the airlines to make further changes.

1.2 Literature Review

Despite the introduction of the CDM paradigm and a greater participation of airlines in the slots allocation procedure, the RBS approach currently in practice, still remains largely centralized today. Fearing and Kash [1] argue that this makes the RBS procedure particularly inefficient since airlines, which have the best visibility into their internal operations costs, are offered very little say in the initial slot allocation process. Thus, several factors like aircraft size, passenger load, mix of connecting passengers and future airline schedules, which would have undoubtedly been considered important had the airlines been involved, are given no consideration at all. Addressing the naivety of the RBS procedure, Hoffman et al. [10] also assert that ideally, the slots allocation procedure should take into consideration the interests of all the concerned stakeholders, namely the FAA (which wants to maximize the system throughput), the airlines (who want to minimize their operating costs) and the passengers (who want on-time performance).

Based on different metrics, several centralized and decentralized rationing procedures have been developed and analyzed in the literature. Ball et al. [11] considers the Ration By Distance and Equity-based Ration By Distance (E-RBD) approaches for allocating arrival slots. In these approaches, long-haul flights are given priorities over short-haul flights while assigning slots at a reduced capacity airport. The rationale for using these approaches is that short-haul flights are able to adapt more quickly to variations in airport arrival rates. Thus, transferring delays to them can lead to a more efficient utilization of the reduced capacity airports. Manley [12] simulates the effects of five different rationing strategies (Ration-by-Distance, Ration-by-

Passengers, Ration-By-Aircraft Size, Ration-By-Fuel Flow High Preference, Ration-By-Fuel Flow Low Preference) on the performance and equity of airlines and passengers at three different airports (JFK, LGA and EWR). Depending on the different criteria, the author finds different optimal strategies for each airport. In addition to these centralized procedures, different rationing schemes based on market-based mechanisms like slot exchanges, slot auctions and congestion pricing have also been explored. Vossen and Ball [9] propose a slot exchange framework in which the FAA acts as a mediator to conduct slots trades between airlines on the day of operations. This kind of a framework provides airlines with a variety of options to select more desirable slots for themselves based on their operational costs. However, because of the complexities involved, it is very difficult to implement such a framework in practice. The idea of slot auctioning particularly has been discussed in great deal in the literature (Rassenti et al. [13], Cholankeril [14], Harsha [15], Plott et al. [16]). Despite its huge popularity in the academic literature, the implementation of slot auctions has met with huge resistance from airlines. The most significant reason for this, as discussed in Ball et al. [17], is that an auction would require airlines to bid money at airports to gain slots. This, in turn, would impose an additional financial burden on the airlines. Mechanisms involving congestion pricing (Brueckner [18], Brueckner [19]) also draw a similar negative response from the airlines for the same reason.

We now describe briefly about the slot rationing procedure developed by Fearing and Kash [1] which will be the main focus of our attention for the remainder of this thesis, and compare it with the existing approaches discussed in this section. In the strategic prioritization framework proposed by Fearing and Kash [1], the airlines participate in a non-monetary auction in which they bid for priority minutes at different airports with a fixed budget of points allocated to them by the FAA. Whenever disruption occurs, the airlines use these priority minutes to schedule their flights ahead of other flights in the initial flight ordering generated for performing RBS. In this scheme, the slots are rationed by a procedure known as Ration By Prioritized Schedule, which is almost identical to the Ration By Schedule procedure, currently in practice.

We see that this kind of a strategic prioritization scheme has several advantages. Firstly, unlike the centralized rationing procedures, in this framework, the airlines have much more control over their allocated schedules as they can prioritize different airports based on their needs. Secondly, in this scheme, the auctions are designed to take place much in advance of the day of flight operations. Thus, in contrast to the slot exchange mechanisms, the complex decisions are taken several (around 6-12) months earlier and hence, the complications associated with slot exchange procedures is avoided. Thirdly and most importantly, in this approach, the airlines do not place monetary bids; rather, they participate in the auctions with bids in the form of points allocated to them by the FAA. This ensures that additional monetary costs are not incurred by the airlines. In addition, once the priority minutes have been allocated to the airlines, the slots allocation procedure is almost identical to the current RBS procedure in practice, making this framework comparatively easier to implement. Another advantage of adopting this framework from a regulator's perspective is that by controlling the points budget of airlines, the regulator can incentivize them to schedule lesser number of flights. This is especially especially useful as it can help in controlling congestion within the air transportation system.

1.3 Contribution and Outline of Thesis

As described in the previous section, the strategic prioritization framework proposed by Fearing and Kash [1] has several practical advantages over other approaches discussed in literature. The main focus of the thesis, therefore, is to develop further the theoretical results for this framework and show through simulations on historical data that significant congestion cost benefits are achieved by using strategic prioritization. The list given below explain the main contributions of the thesis in more detail:

1. *Development of the Modified Ration By Prioritized Schedule (RBPS) Algorithm*

   In this thesis, we first show the limitations of the RBPS procedure introduced by Fearing and Kash [1] and then develop a new, more consistent Modified RBPS algorithm for rationing arrival slots based on prioritized schedules. Using this algorithm, we further construct a comprehensive example to motivate the benefits of adopting a strategic prioritization framework in the real world.

2. *Extension of Theoretical Results*

   We extend the theoretical work done by Fearing and Kash [1] to prove the existence of a pure strategy Nash equilibrium in the second stage of their model for a generalized multiple airlines and multiple airports case. In addition, by showing that the diagonal strict concavity condition (Rosen [20]) holds for the second stage subgame, we also prove that this equilibrium is unique for the two airlines and multiple airports case.

3. *Simulating the Strategic Prioritization Framework for Historical Flight Data*

   We develop two different bidding strategies for the airlines to carry out simulations on historical disruption scenarios. While one strategy simply sets the airline bids at an airport to be proportional to the number of seats on all the incoming flights, the second strategy uses a metric that integrates the concept of delay multipliers (Beatty et al. [21]) with the Airline Disruption Response model (Fearing and Barnhart [22]) to measure delay propagation for an airline at an airport. We then simulate the proposed framework for these airline strategies to investigate the congestion cost benefits of strategic prioritization.

The organization of the thesis is as follows. Chapter 2 gives an overview of the strategic prioritization framework and develops motivation for the approach through an example. Chapter 3 describes the two stage game theoretic model proposed by Fearing and Kash [1] and provides mathematical proofs for the existence and uniqueness of Nash equilibrium in the second stage of the model. Chapter 4 discusses the framework for conducting simulations and develops two different prioritization strategies for airlines. Chapter 5 summarizes the results and limitations of our experiments, and discusses the directions for future research.

Chapter 2

The Strategic Prioritization Approach

In the Introduction chapter, we briefly described about the strategic prioritization framework proposed by Fearing and Kash [1] and discussed its merits over other approaches that have been developed so far. In this chapter, we first provide a more detailed overview of this framework. We then describe the Ration By Prioritized Schedule (RBPS) procedure which Fearing and Kash [1] developed in their paper for rationing prioritized schedules. We show the limitations of this procedure and develop a new Modified RBPS algorithm. Finally, using this new algorithm, we motivate the benefits of strategic prioritization approach through a simple example.

2.1 Strategic Prioritization Framework

The strategic prioritization framework can be basically thought of as consisting of a non-monetary auction in addition to the three-stage sequential evaluation procedure described in Section 1.1.2. As discussed in the last chapter, the key thing to note over here is that the auction for priority minutes takes place much in advance (around 6-12 months) of the sequential evaluation procedure, which is brought into effect on the day of operations whenever a disruption occurs. Each airline first participates in the non-monetary auction, where it bids for priorities at all airports using a pool of points provided by the regulator. For every airport, each airline receives priority minutes from the auction, which are split equally among all its flights scheduled at that airport. The scheduled arrival times of the flights are offset (earlier) by the number of priority minutes their respective airlines receive for those flights. In the subsequent step, the slots are rationed based on the prioritized schedules using a procedure similar to the RBS approach.

2.2 Rationing Prioritized Schedules

Fearing and Kash [1] developed a Ration By Prioritized Schedule (RBPS) procedure for allocating slots to airlines based on prioritized schedules. This RBPS procedure is basically an extension of the RBS procedure currently in practice. In the RBPS procedure, the flights are first arranged in the increasing order of their prioritized times, and then RBS is applied on these times. For clarity (using the same terminology as used by the authors), if a flight f is originally scheduled at a time α_f and it receives a priority of p_f minutes from the auction, then the prioritized time for that flight is given as α_f - p_f. For the implementation of RBPS, the prioritized times, α_f - p_f for all flights are first sorted in increasing order and then, RBS is executed on them. To see how RBPS works, we consider the same hypothetical example that we considered in Section 1. This time, however, we assume that Airlines A and B receive priorities of 4 and 12 minutes respectively for all of their flights. The prioritized schedule, thus created, is given in column 4 of Table 2.1. When RBS is used on prioritized times listed in this column, we get the RBPS schedule as given in column 5 of Table 2.1.

Flight	Original Schedule	RBS	Prioritized Schedule	RBPS
A1	7:00 PM	7:00 PM	6:56 PM	7:00 PM
A2	7:06 PM	7:10 PM	7:02 PM	7:30 PM
B1	7:10 PM	7:20 PM	6:58 PM	7:10 PM
B2	7:12 PM	7:30 PM	7:00 PM	7:20 PM

Table 2.1: Original, RBS, Prioritized and RBPS schedules for a controlled airport with reduced capacity of 1 flight per 10 min

In the above example, we see that by gaining priority minutes, the airline B flights (B1 and B2) are now scheduled ahead of the airline A flight-A2, which was originally scheduled earlier than both the airline B flights. If we consider the example more carefully, however, we see a caveat that is not discussed in this example and was also not discussed explicitly by Fearing and Kash [1] in their paper. The problem with RBPS procedure is that it can potentially allocate new arrival slots, which are scheduled earlier than the original scheduled arrival time. Since flights cannot arrive before their original times, in such cases, the allocated arrival slots cannot possibly be utilized. To see how this can happen, we consider the same original schedule that we considered in Table 2.1. However, we now modify the priorities received by airlines A and B from 4 and 12 minutes respectively to 2 and 15 minutes respectively. The new prioritized times and the corresponding RBPS times are then given in columns 4 and 5 of Table 2.2 respectively. In this example, we see that the RBPS arrival times of flights B1 and B2 (7:00 PM and 7:10 PM respectively) are earlier than their original scheduled arrival times (7:10 PM and 7:12 PM respectively) and hence, we cannot necessarily assume that these slots can be used by flights B1 and B2.

Drawing motivation from the above example, we now try to modify the existing

Flight	Original Schedule	RBS	Prioritized Schedule	RBPS
A1	7:00 PM	7:00 PM	6:58 PM	7:20 PM
A2	7:06 PM	7:10 PM	7:04 PM	7:30 PM
B1	7:10 PM	7:20 PM	6:55 PM	7:00 PM
B2	7:12 PM	7:30 PM	6:57 PM	7:10 PM

Table 2.2: Pathological Example for RBPS: Original, RBS, Prioritized and RBPS Schedules for a controlled airport with reduced capacity of 1 flight per 10 min

RBPS approach so that it can be used even for such pathological cases. In our Modified RBPS algorithm, we first arrange the flights in a list in the increasing order of their prioritized times, α_f-p_f. We then traverse the list in the same order and search for the first unallocated available time slot at or after the scheduled arrival time of the flight and assign the flight to that time slot. We illustrate the implementation of this algorithm through the example that we considered in Table 2.2. Based on the Prioritized Schedule column in Table 2.2, we first create Table 2.3, in which rows are sorted in the increasing order of prioritized times. Going from top to bottom in this table, the algorithm assigns the first available slot at or after the original scheduled arrival time (given in column 3 of Table 2.3) to each flight. We therefore get the new Modified RBPS schedule as shown in column 4 of Table 2.3. Comparing this with the RBPS schedules given in columns 5, we see that there are considerable differences between the two allocated schedules. This is because the RBPS assigns slots in a manner that preserves the order of the prioritized flight times. On the other hand, the Modified RBPS uses the prioritized schedules to ensure that the original scheduled arrival times can be assigned to the prioritized flights.

Flight	Prioritized	Original	Modified RBPS	RBS	RBPS
B1	6:55 PM	7:10 PM	7:10 PM	7:20 PM	7:00 PM
B2	6:57 PM	7:12 PM	7:20 PM	7:30 PM	7:10 PM
A1	6:58 PM	7:00 PM	7:00 PM	7:00 PM	7:20 PM
A2	7:04 PM	7:06 PM	7:30 PM	7:10 PM	7:30 PM

Table 2.3: Modified RBPS algorithm for a Controlled Airport with reduced capacity of 1 flight per 10 min

2.3 Motivation for Prioritization - An Example

While the fairness criterion of first-scheduled, first-served is upheld in the RBS scheme, there is a considerable loss of overall system efficiency in this approach. To demonstrate this, we consider a simple example consisting of 2 airlines (A and B) and 2 airports (X and Y). In this example, for simplicity, we take into account only two types of delays: 1) The flight delays associated with the late arrival of an aircraft, and 2) Passenger delays due to missed connections. We disregard any other types of possible delays like crew delays, baggage delays, etc.

Airline A is representative of a large low cost, point-to-point carrier like Southwest Airlines and has a seating capacity of 120, while Airline B represents a legacy carrier like American Airlines, which uses a hub-and-spoke network model and has a seating capacity of 200. Tables 2.4 and 2.5 list out typical schedules of airlines A and B respectively at a major, non-hub airport X. Observing the schedules, it can be easily inferred that from the perspective of making connections, Airline A has a much tighter schedule as compared to Airline B at Airport X.

Table 2.6 shows the arrival schedule at Airport X, obtained by combining the arrival schedules for Airlines A and B for the time interval between 5:25 PM to 5:40

Aircraft	Arrival	Next flight departure
A1	5:25 PM	6:25 PM
A2	5:30 PM	6:25 PM
A3	5:32 PM	6:40 PM

Table 2.4: Arrival Times of first flights and departure times of next flights for airline A aircrafts

Aircraft	Arrival	Next flight departure
B1	5:25 PM	10:50 PM
B2	5:29 PM	10:27 PM

Table 2.5: Arrival times of first flights and departure times of next flights for airline B aircrafts

PM. In the event of any disruptions due to bad weather, it is assumed that the arrival rate for Airport X is reduced to 1 flight/10 min and Ration by Schedule is used to allocate time slots to the flights. This allocation is detailed in the RBS column of Table 2.6. Assuming that a minimum of 45 min of time difference is required between two consecutive flight legs for an aircraft, if the original RBS schedule is used, it can be seen that there will be a **total delay of 1 hour and 49 min – 40 min for Aircraft A2** (=25 min for first flight leg +15 min for second flight leg) corresponding to its late 5:55PM arrival, **43 min delay for Aircraft A3** (=33 min for first flight leg+10 min for second flight leg) corresponding its late 6:05 PM arrival, **10 min for Aircraft B1** and **16 min for Aircraft B2**. We note that for both Aircrafts B1 and B2, their entire delays correspond only to the first flight legs and there are no delays in the second flight legs since the time difference between the first and second flight legs is more than one hour. We now summarize the calculations are summarized as under:

Aircraft	Scheduled Arrival	RBS	Prioritized Schedule	RBPS
A1	5:25 PM	5:25 PM	5:19 PM	5:25 PM
B1	5:25 PM	5:35 PM	5:25 PM	5:45 PM
B2	5:29 PM	5:45 PM	5:29 PM	6:05 PM
A2	5:30 PM	5:55 PM	5:24 PM	5:35 PM
A3	5:32 PM	6:05 PM	5:26 PM	5:55 PM

Table 2.6: Combined Original, Prioritized, RBS and RBPS schedules for Airlines A and B at Airport X

Flight Delays Associated with RBS Allocation at Airport X

Total delay for Airline A= 83 min (=40 min for A2 + 43 min for A3)

Total delay for Airline B= 26 min (=10 min for B1 + 16 min for B2)

Overall airline delays at Airport X= 83 min + 26 min = 109 min

Passenger Delays Associated with RBS Allocation at Airport X

Total passenger delays for airline A =120*83=9960 min= 166 hours

Total passenger delays for airline B =200*26=5200 min =86.66 hours

Overall passenger delays at Airport X =252.66 hours

It is interesting to note that in this example if the flights had been strategically prioritized, the overall delays could have been significantly reduced. In particular, we consider a case in which the arrival times for the first legs of both Aircrafts A2 and A3 have been scheduled 6 minutes prior to their original arrival times. This causes A2 to be scheduled ahead of B1 and A3 to be scheduled ahead of B2 (see Prioritized Schedule column in Table 3). In this case, if RBS is implemented on the new Prioritized schedule to meet the reduced capacity constraints at Airport X, then the propagated delay in the second flight legs for both A2 and A3 is avoided (see RBPS column in Table 3) and the overall delay is reduced to **1 hr and 24 min** – **5 min for Aircraft A2**, **23 min for Aircraft A3**, **20 min for Aircraft B1** and **36 min for Aircraft B2**. We further see from the calculations below that the total passenger delays are also reduced.

Flight Delays Associated with RBPS Allocation at Airport X

Total delay for Airline A= 28 min (5 min for A2 + 23 min for A3)

Total delay for Airline B= 56 min (20 min for B1 +36 min for B2)

Overall airlines delay at Airport X= 28 min + 56 min = 84 min

Passenger Delays Associated with RBPS Allocation at Airport X

Total passenger delay for airline A (RBPS) =120 *28 min= 3360 min = 56 hours

Total passenger delay for airline B (RBPS) = 200*56=11200 min=186.66 hours

Overall passenger delay (RBPS) at Airport X = 242.66 hours

We thus see from this example, that by trading off fairness for system efficiency, the delays in the system can be reduced. However, there should be some incentive for Airline B (a legacy carrier) to conform to this (unfair) RBPS schedule. In order to develop this idea further, we consider another Airport Y which is assumed to be a hub for Airline B. At any typical airline hub, it is of paramount importance for that airline to maintain its schedule as much as possible. This is because at a hub, in contrast to point-to-point carriers, a legacy carrier has a majority of its air traffic consisting of passengers connecting to other airports rather than completing their journey at Airport Y itself. Thus, any delays in flight arrivals at hubs result in missed connections for the passengers or delays in departures for subsequent connecting flights. The usefulness of strategic prioritization in such a case is illustrated by further extending our example.

Table 2.7 lists the arrival times of flights along with their origins for Airline B (legacy carrier). We assume that all Airline B flights listed in this table have a capacity of 200 passengers each - of which 80 percent passengers are connecting to other locations while 20 percent are completing their journeys at Airport Y. We further assume that the 80 percent passengers (on each flight) who are connecting to other locations are split up equally amongst Airline B flights in Table 2.9.

Flight	**Origin**	**Arrival**
B1	Minneapolis (MSP)	9:55 AM
B2	Washington (DCA)	9:55 AM
B3	Chicago (MDW)	9:55 AM

Table 2.7: Airline B (legacy carrier) incoming flights- with origin and arrival times.

Flight	**Origin**	**Arrival**
A1	Oklahoma City (OKC)	9:45 AM
A2	Pittsburgh (PIT)	9:50 AM

Table 2.8: Airline A (point to point carrier) incoming flights- with origin and arrival times

Whenever there are bad weather conditions at Airport Y, assuming again that the airports flight arrival capacity is reduced to 1 flight/10 min, RBS is used for generating the new schedule. This schedule is described in Table 2.10. We see that if the original order is maintained and a minimum connecting time of 30 min

Flight	Destination	Arrival
B4	Austin (AUS)	10:35 AM
B5	Houston (HOU)	10:40 AM
B6	New Orleans (MSY)	10:45 AM
B7	Dallas (DAL)	10:45 AM
B8	Las Vegas (LAS)	10:48 AM

Table 2.9: Connections for Airline B incoming flights listed in Table 2.7.

is assumed, then all the passengers from the incoming flight B3 (in Table 2.7) will either miss their connecting flights (if the connecting flights departed on time) or the connecting flights will have to be delayed to an earliest possible departure time of 10:55 AM in order to allow everyone to make their connections. From the calculations below, the total airlines delay= **125 min** (5 min for Airline A+120 min for Airline B) and total passenger delay= **164.66 hours** (16.66 hours for Airline A passengers and 148 hours for Airline B passengers) in this scenario.

Flight	Destination	Arrival	RBS	Prioritized	RBPS
A1	Oklahoma City (OKC)	9:45 AM	9:45 AM	9:45 AM	9:45 AM
A2	Pittsburgh (PIT)	9:50 AM	9:55 AM	9:50 AM	10:25 AM
B1	Minneapolis (MSP)	9:55 AM	10:05 AM	9:49 AM	9:55 AM
B2	Washington (DCA)	9:55 AM	10:15 AM	9:49 AM	10:05 AM
B3	Chicago (MDW)	9:55 AM	10:25 AM	9:49 AM	10:15 AM

Table 2.10: Combined Original, Prioritized, RBS and RBPS schedules for Airlines A and B at Airport Y

However, we see that if 6 minutes of priority is assigned to each incoming flight of Airline B (as described in Prioritized Schedule Column of Table 4) and RBPS is implemented, the total delay is reduced to **80 min** (=35min for Airline A and 45 min for Airline B).

Flight Delays Associated with RBS Allocation at Airport Y

Total delay for Airline A = **5 min** (for A2)

Total delay for Airline B = 10 (for B1) + 20 (for B2) + 30 (for B3) + 20 (for B4) + 15 (for B5)+ 2*10 (for B6 and B7)+ 5 (for B8)=**120 min**

Overall airlines delay at Airport Y = 5 min + 120 min = **125 min**

Flight Delays Associated with RBPS Allocation at Airport Y

Total delay for Airline A = **35 min**

Total delay for Airline B = 10 (for B2) + 20 (for B3) + 10 min (for B4) + 5 min (for B5) = **45 min**

Overall airlines delay at Airport Y = 35 min + 45 min = **80 min**

Passenger Delays Associated with RBS Allocation at Airport Y

Total passenger delay for Airline A=5*200=**16.66 hours** (1000 min)

Total passenger delay for Airline B = (0.2*(10+20+30) + 20 (for B4) + 15 (for B5) + 2*10 (for B6 and B7) + 7min (forB8))*120 = **148hours** (8760 min)

Overall passenger delay at Airport Y = 148+16.66 =**164.66 hours**

Passenger Delays Associated with RBPS Allocation at Airport Y

Total passenger delay for Airline A = 35*200 min =**116.66 hours** (7000 min)

Total passenger delay for Airline B = (0.2*(10+20)+10min (for B4) +5min (for B5)) * 120 = **42 hours** (2520 min)

Overall passenger delay at Airport Y = 116.66+42 = **158.66 hours**

Combining the airline delays on both airports X and Y, we see that strategic prioritization by airlines reduced the total airline delays by **70 min** (25 min for

Airport X + 45 min for Airport Y). Similarly, the total passenger delays are reduced by **16 hours** (10 hours at Airport X and 6 hours at Airport Y). When we consider the total delay from the individual airlines perspective, we see that strategic prioritization reduces the total delay for Airline A by **25 min** (55 min gained at Airport X and 30 min lost at Airport Y) and for Airline B by **45 min** (75 min gained at Airport Y and 30 min lost at Airport X). Thus, overall both the airlines do better.

Chapter 3

Model for Prioritization and Theoretical Results

In our discussion so far, we have considered the merits of prioritizing flights across different airports and showed through a simple example how this could potentially help in reducing both airline as well as passenger delays. However, till now, we have not yet discussed how the priorities should be allocated to airlines in practice. In this chapter, therefore, we first describe formally the two-stage game-theoretic model, developed by Fearing and Kash [1], that provides the framework for allocating priority minutes to airlines. We further extend the theoretical results for this model to prove that a pure strategy Nash equilibrium exists in the second stage of the model for the generalized multiple airlines and multiple airports case. In addition, we also prove that this equilibrium is unique for a reduced case in which there are only two airlines.

3.1 Priority Allocation to Airlines

In this section, we describe the two stage game theoretic model proposed by Fearing and Kash [1] for allocating priority minutes to airlines at different airports.In the first stage of this model, each airline a makes decisions about the number of flights N_{ra} it wants to schedule at each airport r. In the second stage, all the airlines participate in a non-monetary auction, in which they bid points (instead of money) from a budget of total B_a points allocated to them by the FAA. In our model, we assume B_a to be proportional to the total number of flights ($=\sum_r N_{ra}$) scheduled by airline a. The airlines receive priority minutes proportional to the number of points b_{ra} they bid at each airport. These priority minutes are then divided equally among all the N_{ra} flights scheduled at that airport. Thus, to summarize, the priority p_{rai} received by an airline a at airport r for a flight i scheduled at that airport is given by the following expression.

$$p_{rai} = \frac{P_r}{N_{ra}} \frac{b_{ra}}{\sum_{a'} b_{ra'}} \tag{3.1}$$

In the above expression, the bids b_{ra} are subject to the constraints $\sum_r b_{ra} \leq B_a$ $\forall a$. We note that these constraints compel the airlines to make prioritization trade-offs across airports.

For describing the payoffs of airlines (players) in this game, the authors assume that each airline receives a fixed positive utility value V for scheduling a flight and incurs a disutility cost (equal to the delay) for acquiring delays on the scheduled flights. In accordance with their original RBPS procedure, Fearing and Kash [1] consider the flight's delays or disutility costs to be a step function of the flight's priority minutes. To see why this is true, consider the scheduled arrival time of flight i of airline a at airport r to be a uniformly distributed random variable $\alpha_{rai} \in [0, T_r]$. The original RBPS scheduled time $\widetilde{\alpha}_{rai}$ of the flight is then calculated in terms of the fixed slot size z_r as

$$\widetilde{\alpha}_{rai} = z_r \sum_{(r,a',j) \neq (r,a,i)} \mathbb{I}(\alpha_{ra'j} - p_{ra'j} \leq \alpha_{rai} - p_{rai}) \tag{3.2}$$

Consequently, the delay d_{rai} is defined as $d_{rai} = \alpha_{rai} - \widetilde{\alpha}_{rai}$. From the Eq. 3.2, it is quite clear that the discontinuities in the delay step function arise whenever a flight gains sufficient priority minutes to get scheduled ahead of another flight. In order to avoid the analytical difficulties involved in dealing with step functions, the model uses the expected utility value as payoffs for the airlines, i.e., it assumes the payoff u_{rai} of airline a for scheduling flight i at airport r to be

$$u_{rai} = \mathbb{E}_{\alpha_{\mathbf{r}}}[V - \lambda_r d_{rai}] \tag{3.3}$$

In the above equation, λ_r denotes the probability of bad weather at airport r. The total payoffs v_a for an airline a can thus be calulated as $v_a = \sum_r \sum_{i=1}^{N_{ra}} u_{rai}$. The idea of taking α_{rai} to be a uniform random variable and expressing the payoffs u_{rai} to be expected utility values is justified on the basis that the airlines make their bidding decisions on a strategic time scale (i.e. about 6-12 months in advance) and by that time the schedules of the flights are usually not finalized. Thus, the scheduled arrival times α_{rai} are assumed to have a uniform distribution. In addition, taking the payoffs u_{rai} to be expected utility values simplifies the complications introduced in the model due to discontinuous delay functions.

3.2 Theoretical Results

Having discussed the framework of the model, we are now in a position to describe our results. Fearing and Kash [1] showed the existence of a pure strategy subgame perfect Nash equilibrium for a reduced game with 2 airlines and multiple airports. In this section, we follow a similar approach to extend their results of existence to the more general case with multiple airlines and multiple airports. In addition, we also derive the conditions for the uniqueness of the equilibrium using the diagonal strict concavity condition (Rosen [20]) for the reduced game with 2 airlines and multiple airports.

3.2.1 Existence of a Pure Strategy Nash equilibrium

For proving the existence of a pure strategy Nash equilibrium in the second stage subgame, we use the existence conditions proposed by Debreu [23], Glicksberg [24] and Fan [25]. These conditions state that for a strategic-form game $< I, (S_i), (u_i) >$, a pure strategy Nash equilibrium exists if the strategy sets S_i are non-empty, compact and convex, and the payoff functions u_i are continuous in $\boldsymbol{s}$ and quasi-concave in s_i.

Since for our game theoretic model, the bids b_{ra} of the airlines are subject to non-negativity constraints, $b_{ra} \geq 0$ and budget constraints, $\sum_r b_{ra} \leq B_a$, therefore, the strategy sets of the airlines for the second stage subgame are both closed and bounded (and hence, compact by definition). To prove that the payoff functions $v_a = \sum_r \sum_{i=1}^{N_{ra}} u_{rai}$ are continuous in $\boldsymbol{s}$ and concave s_i, we first simplify the expression for u_{rai}.

$$
\begin{aligned}
u_{rai} &= \mathbb{E}_{\alpha_{\mathbf{r}}}[V - \lambda_r d_{rai}] \\
&= V - \lambda_r \mathbb{E}_{\alpha_{\mathbf{r}}}[d_{rai}] \\
&= V - \lambda_r \mathbb{E}_{\alpha_{\mathbf{r}}}[\widetilde{\alpha}_{rai} - \alpha_{rai}] \\
&= V - \lambda_r (\mathbb{E}_{\alpha_{\mathbf{r}}}[\widetilde{\alpha}_{rai}] - \mathbb{E}_{\alpha_{\mathbf{r}}}[\alpha_{rai}]) \\
&= V - \lambda_r \left(\mathbb{E}_{\alpha_{\mathbf{r}}}[\widetilde{\alpha}_{rai}] - \frac{T_r}{2} \right) \\
&= V + \lambda_r \frac{T_r}{2} - \lambda_r \mathbb{E}_{\alpha_{\mathbf{r}}}[\widetilde{\alpha}_{rai}] \\
&= V + \lambda_r \frac{T_r}{2} - \lambda_r \mathbb{E}\Bigg[z_r \sum_{(r,a',j)\neq(r,a,i)} \mathbb{I}(\alpha_{ra'j} - p_{ra'j} \leq \alpha_{rai} - p_{rai}) \Bigg] \\
&= V + \lambda_r \frac{T_r}{2} - \lambda_r z_r \sum_{(r,a',j)\neq(r,a,i)} \mathbb{E}[\mathbb{I}(\alpha_{ra'j} - p_{ra'j} \leq \alpha_{rai} - p_{rai})] \\
&= V + \lambda_r \frac{T_r}{2} - \lambda_r z_r \sum_{(r,a',j)\neq(r,a,i)} \mathbb{P}(\alpha_{rai} - \alpha_{ra'j} \geq p_{rai} - p_{ra'j}) \\
&= V + \lambda_r \frac{T_r}{2} - \lambda_r z_r \sum_{(r,a',j)\neq(r,a,i)} \mathbb{P}(\alpha_{rai} - \alpha_{ra'j} \geq p_{rai} - p_{ra'j}) \\
&= V + \lambda_r \frac{T_r}{2} - \lambda_r z_r \left(\sum_{j\neq i} \mathbb{P}(\alpha_{rai} \geq \alpha_{raj}) + \sum_{a'\neq a} \sum_{j=1}^{N_{ra'}} \mathbb{P}(\alpha_{rai} - \alpha_{ra'j} \geq p_{rai} - p_{ra'j}) \right) \\
&= V + \lambda_r \frac{T_r}{2} - \frac{\lambda_r z_r}{2}(N_{ra} - 1) - \lambda_r z_r \sum_{a'\neq a} \sum_{j=1}^{N_{ra'}} \mathbb{P}(\alpha_{rai} - \alpha_{ra'j} \geq p_{rai} - p_{ra'j}) \\
&= V + \lambda_r \frac{T_r}{2} - \frac{\lambda_r z_r}{2}(N_{ra} - 1) - \lambda_r z_r \sum_{a'\neq a} \sum_{j=1}^{N_{ra'}} \big(1 - \mathbb{P}(\alpha_{rai} - \alpha_{ra'j} \leq p_{rai} - p_{ra'j})\big) \\
&= V + \lambda_r \frac{T_r}{2} - \frac{\lambda_r z_r}{2}(N_{ra} - 1) - \lambda_r z_r \sum_{a'\neq a} \sum_{j=1}^{N_{ra'}} \big(1 - F(\tau_{raa'}(b_r))\big) \\
&= V + \lambda_r \frac{T_r}{2} - \frac{\lambda_r z_r}{2}(N_{ra} - 1) - \lambda_r z_r \sum_{a'\neq a} N_{ra'} + \lambda_r z_r \sum_{a'\neq a} \sum_{j=1}^{N_{ra'}} F\big(\tau_{raa'}(b_r)\big)
\end{aligned}
\tag{3.4}
$$

For the second stage subgame, we note that that all the terms except the last in the right hand side are constant and thus, for analyzing the second stage game, we can ignore them and define u_{rai} as follows:

$$u_{rai} = \lambda_r z_r \sum_{a' \neq a} \sum_{j=1}^{N_{ra'}} F(\tau_{raa'}(b_r)) \tag{3.5}$$

$$\begin{aligned} v_a &= \sum_r \sum_{i=1}^{N_{ra}} \left(\lambda_r z_r \sum_{a' \neq a} \sum_{j=1}^{N_{ra'}} F(\tau_{raa'}(b_r)) \right) \\ &= \sum_r \sum_{a' \neq a} \sum_{i=1}^{N_{ra}} \sum_{j=1}^{N_{ra'}} \lambda_r z_r F(\tau_{raa'}(b_r)) \\ &= \sum_r \sum_{a' \neq a} N_{ra} N_{ra'} \lambda_r z_r F(\tau_{raa'}(b_r)) \end{aligned} \tag{3.6}$$

Fearing and Kash [1] in their paper derived the analytical expression for the cdf $F(\tau)$ and its derivatives $f(\tau)$ and $f'(\tau)$ as follows:

$$F(\tau) = \begin{cases} \frac{1}{2T_r^2}(T_r^2 + 2T_r\tau - \tau^2) & \tau \geq 0 \\ \frac{(T_r+\tau)^2}{2T_r^2} & \tau < 0 \end{cases}$$

$$f(\tau) = \begin{cases} \frac{T_r-\tau}{T_r^2} & \tau \geq 0 \\ \frac{T_r+\tau}{T_r^2} & \tau < 0 \end{cases}$$

$$f'(\tau) = \begin{cases} \frac{-1}{T_r^2} & \tau > 0 \\ \frac{1}{T_r^2} & \tau < 0 \\ \text{undefined} & \tau = 0 \end{cases}$$

From the above expressions, we see that the payoff functions v_a are continuous in $\boldsymbol{s}$. We now prove that v_a are also concave over airline a's strategy set $S_a = (b_{1a}, ..., b_{ra}, ..., b_{na}) \in \mathbb{R}^n$ of airline a. Since, concavity implies quasiconcavity, therefore, our last requirement for the existence of pure strategy Nash equilibrium will be satisfied.

To prove that v_a is concave, we show that its Hessian is negative semi-definite. Evaluating the double derivatives of v_a with respect to the bids,

$$v_a = \sum_{r'} \sum_{a' \neq a} N_{r'a} N_{r'a'} \lambda_{r'} z_{r'} F(\tau_{r'aa'}(b_{r'})) \tag{3.7}$$

$$\frac{\partial v_a}{\partial b_{ra}} = \sum_{a' \neq a} N_{ra} N_{ra'} \lambda_r z_r \left(f(\tau_{raa''}) \frac{\partial \tau_{raa'}}{\partial b_{ra}} \right) \tag{3.8}$$

$$\frac{\partial^2 v_a}{\partial b_{ra}^2} = \sum_{a' \neq a} N_{ra} N_{ra'} \lambda_r z_r \left(f(\tau_{raa'}) \frac{\partial^2 \tau_{raa'}}{\partial b_{ra}} + f'(\tau_{raa'}) \left(\frac{\partial \tau_{raa'}}{\partial b_{ra}} \right)^2 \right) \tag{3.9}$$

$$\frac{\partial^2 v_a}{\partial b_{ra} b_{r'a}} = 0 \quad (\text{for } r \neq r') \tag{3.10}$$

From Eq. 3.10 we see that all the non-diagonal elements (corresponding to $r \neq r'$) of the Hessian are 0. Therefore, the Hessian is a diagonal matrix. For evaluating $\frac{\partial^2 v_a}{\partial b_{ra}^2}$, the diagonal entry in the Hessian corresponding to the row and column of bid b_{ra}), we calculate the relevant terms required in Eq. 3.9.

$$\tau_{raa'} = \frac{P_r}{\sum_{a''} b_{ra''}} \left(\frac{b_{ra}}{N_{ra}} - \frac{b_{ra'}}{N_{ra'}} \right) \tag{3.11}$$

$$\frac{\partial \tau_{raa'}}{\partial b_{ra}} = \frac{P_r}{\left(\sum_{a''} b_{ra''} \right)^2} \left(\frac{\sum_{a'' \in A - \{a\}} b_{ra''}}{N_{ra}} + \frac{b_{ra'}}{N_{ra'}} \right) \tag{3.12}$$

$$\frac{\partial \tau_{raa'}}{\partial b_{ra}} = -\frac{2P_r}{\left(\sum_{a''} b_{ra''} \right)^3} \left(\frac{\sum_{a'' \in A - \{a\}} b_{ra''}}{N_{ra}} + \frac{b_{ra'}}{N_{ra'}} \right) \tag{3.13}$$

Case 1: When $\tau_{raa'} > 0$

In this case, $f'(\tau_{raa'}) = -\frac{1}{T_r^2} < 0$ and since, $f(\tau_{aa'}) \geq 0$, $\frac{\partial^2 \tau_{raa'}}{\partial b_{ra}} < 0$ and $\frac{\partial \tau_{raa'}}{\partial ra} \geq 0$. Therefore,

$$f(\tau_{raa'})\frac{\partial^2\tau_{raa'}}{\partial b_{ra}} + f'(\tau_{raa'})\bigg(\frac{\partial\tau_{raa'}}{\partial b_{ra}}\bigg)^2 \leq 0 \tag{3.14}$$

Case 2: When $\tau_{raa'} < 0$

We now assume $\sum_{a''} b_{ra''} = k$ and $\left(\frac{\sum_{a''\in A-\{a\}} b_{ra''}}{N_{ra}} + \frac{b_{ra'}}{N_{ra'}}\right) = y$. Thus, Eq. 3.9 can be written as

$$\begin{aligned}
f(\tau_{raa'})\frac{\partial^2\tau_{raa'}}{\partial b_{ra}} + f'(\tau_{raa'})\bigg(\frac{\partial\tau_{raa'}}{\partial b_{ra}}\bigg)^2 &= \bigg(\frac{T_r + \tau_{raa'}}{T_r^2}\bigg)\bigg(\frac{-2P_r y}{k^3}\bigg) + \bigg(\frac{1}{T_r^2}\bigg)\bigg(\frac{P_r y}{k^2}\bigg)^2 \\
&= \frac{P_r y}{k^3 T_r^2}\bigg(-2(T_r + \tau_{raa'}) + \frac{P_r y}{k}\bigg)
\end{aligned}$$

Expanding the term within brackets on the right hand side,

$$\begin{aligned}
&-2(T_r + \tau_{raa'}) + \frac{P_r y}{k} \\
&\quad = -2\bigg(T_r + \frac{P_r}{k}\bigg(\frac{b_{ra}}{N_{ra}} - \frac{b_{ra'}}{N_{ra'}}\bigg)\bigg) + P_r\bigg(\frac{\sum_{a''\in A-\{a\}} b_{ra''}}{N_{ra}} + \frac{b_{ra'}}{N_{ra'}}\bigg) \\
&\quad = \bigg(-2T_r - \frac{2P_r}{N_{ra}}\bigg)b_{ra} + \bigg(-2T_r + \frac{3P_r}{N_{ra'}} + \frac{P_r}{N_{ra}}\bigg)b_{ra'} \\
&\qquad + \bigg(-2T_r + \frac{P_r}{N_{ra}}\bigg)\sum_{a''\in A-\{a,a'\}} b_{ra''}
\end{aligned}$$

Since the bids for airlines are always non-negative, thus, if both $-2T_r + \frac{P_r}{N_{ra}} \leq 0$ and $-2T_r + \frac{3P_r}{N_{ra'}} + \frac{P_r}{N_{ra}} \leq 0$, then $f(\tau_{raa'})\frac{\partial^2\tau_{raa'}}{\partial b_{ra}} + f'(\tau_{raa'})\bigg(\frac{\partial\tau_{raa'}}{\partial b_{ra}}\bigg)^2 \leq 0$.

We notice that for the first condition to hold $P_r \leq \frac{T_r}{2}min(N_{ra}, N_{ra'})$ is a sufficient condition. The second condition $P_r \leq 2T_r N_{ra}$, then becomes redundant if the first condition is true. From Eq. 3.9, since the condition must be true for all $a' \in A - \{a\}$, therefore the sufficient condition for the Hessian to be negative semi-definite or equivalently, for v_a to be concave is that $P_r \leq \frac{T_r}{2}min_{a'\in A}\{N_{ra'}\}\ \forall r$.

3.2.2 Uniqueness of the Nash Equilibrium

Rosen [20] proved that a strategic-form game $< I, (S_i), (u_i) >$ with strategy sets $S_i = \{x \in \mathbb{R}^{m_i} | h_i(x_i) \geq 0\}$ has a unique pure strategy Nash equilibrium, if the following conditions hold true:

1. $h_i(x_i)$ are concave, which implies that the strategy sets S_i are convex sets.
2. $\exists\ \bar{x} \in \mathbb{R}^{m_i}$ such that $h_i(\bar{x}_i) > 0$.
3. The payoff functions $(u_1, ..., u_n)$ are diagonally strict concave, i.e., they satisfy the condition:

   $$(\bar{x} - x^*)^T \nabla u(x^*) + (x^* - \bar{x})^T \nabla u(\bar{x}) > 0$$

   In the above condition,

   $\nabla u(x) = [\nabla_1 u_1(x), ..., \nabla_n u_n(x)]^T$ and $\nabla_i u_i(x) = \left[\frac{\partial u_i(x)}{\partial x_i^1}, ..., \frac{\partial u_i(x)}{\partial x_i^{m_i}}\right]^T$

Let $U(x)$ denote the jacobian of $\nabla u(x)$. Then, a sufficient condition for proving that the game is diagonally strict concave is to show that $(U(x) + U^T(x))$ is a negative semidefinite matrix. In our model, we assume that there are m airlines and n airports. The strategy set of each airline consists of n bids, in which each bid corresponds to some airport. Thus, for our model, $\nabla v(\mathbf{b})$ is a vector of length mn defined as:

$$\nabla v(\mathbf{b}) = \left[\frac{\partial v_1(\mathbf{b})}{\partial b_{11}}, ..., \frac{\partial v_1(\mathbf{b})}{\partial b_{n1}}, ..., \frac{\partial v_m(\mathbf{b})}{\partial b_{1m}}, ..., \frac{\partial v_m(\mathbf{b})}{\partial b_{nm}}\right]^T$$

The jacobian of $\nabla v(\mathbf{b})$, represented as $V(\mathbf{b})$, is thus an mn x mn matrix. The element in the matrix $(V(\mathbf{b}) + V^T(\mathbf{b}))$ corresponding to the row for bid b_{ra} and column for the bid $b_{r'a'}$ can then be calculated as $\frac{\partial^2}{\partial b_{ra} \partial b_{r'a'}}[v_a + v_{a'}]$.

3.2.2.1 Two airlines case

We next derive the conditions for the uniqueness of Nash equilibrium for the reduced game with 2 airlines (a and a') and multiple airports. In this case, the payoff functions v_a and $v_{a'}$ in Eq. 3.6 simply reduces to

$$\begin{aligned} v_a &= \sum_r N_{ra} N_{ra'} \lambda_r z_r F(\tau_{raa'}(b_r)) \\ v_{a'} &= \sum_r N_{ra} N_{ra'} \lambda_r z_r F(\tau_{ra'a}(b_r)) \end{aligned}$$

We note from above that

$$\begin{aligned} v_a + v_{a'} &= \sum_r N_{ra} N_{ra'} \lambda_r z_r (F(\tau_{raa'}(b_r)) + F(\tau_{ra'a}(b_r))) \\ &= \sum_r N_{ra} N_{ra'} \lambda_r z_r \end{aligned}$$

The last equation comes from the observation that $F(\tau_{raa'}(b_r)) + F(\tau_{ra'a}(b_r)) = 1$.

For evaluating each entry $\frac{\partial^2}{\partial b_{ra} \partial b_{r'a'}}[v_a + v_{a'}]$ of the matrix $(V(\mathbf{b}) + V^T(\mathbf{b}))$ for this reduced game, we consider the following cases:

1. $\boldsymbol{r \neq r'}$: In this case, entry is 0.

2. $\boldsymbol{r = r'}$ **and** $\boldsymbol{a \neq a'}$ **:** In this case, $v_a + v_{a'} = \sum_r N_{ra} N_{ra'} \lambda_r z_r =$ a constant (independent of bids). Therefore, the entry $\frac{\partial^2}{\partial b_{ra} \partial b_{r'a'}}[v_a + v_{a'}]$ for this case also will be 0 in the matrix.

3. $\boldsymbol{r = r'}$ **and** $\boldsymbol{a = a'}$ **:** In this case, $\frac{\partial^2}{\partial b_{ra} \partial b_{r'a'}}[v_a + v_{a'}] = \frac{\partial^2 (2 v_a)}{\partial b_{ra}^2}$.

Fearing and Kash [1] proved that v_a is strictly concave in b_{ra} under the condition $P_r \leq \frac{T_r}{2} min(N_{ra}, N_{ra'})$. Thus, we see that if this condition holds for $\forall r$, then for the two airline case, $(V(\mathbf{b}) + V^T(\mathbf{b}))$ is basically a diagonal matrix with all elements on the diagonal less than 0. Hence, $(V(\mathbf{b}) + V^T(\mathbf{b}))$ is a negative semi-definite matrix.

Chapter 4

Framework and Airline Strategies for Simulations

In the previous chapters, we described how the central regulator can use a bidding procedure to assign priorities to the airlines, and further discussed about the RBPS mechanism for allocating arrival slots to airlines based on prioritized schedules. In order to perform our simulations, we also need a framework to determine the responses of the airlines once the slots have been allocated to them. For this, we first introduce the Airline Disruption Response Model developed by Fearing and Barnhart [22]. We then describe the strategies used by airlines for determining their bids in our simulations. Lastly, we discuss the pre-processing steps involved for generating scenarios and aircraft routings for the simulations.

4.1 Airline Disruption Response Model

To determine the airline responses for our simulations after new arrival slots have been allocated to them by the RBPS procedure, we make use of a simplified version of the Airline Disruption Response model developed by Barnhart et al. [7] and Fearing and Barnhart [22]. This model allows airlines to make recovery decisions by optimizing their schedules through flight swaps and cancellations within their allocated slots. The most significant simplification that we make in the model for performing our simulations is that due to the unavailability of data, we do not explicitly take into consideration the itineraries of passengers.

Next, we explain the elements of the Airline Disruption Response model in more detail and discuss some of the simplifications we have made in the model for carrying out our simulations. In the model, the day of operations is divided into a set $\mathcal{T}$, consisting of discrete time intervals, $\{0, 1, 2, ..., T-1\}$. For our experiments, we assume that each of these discrete time intervals correspond to a duration of five minutes. The set $\mathcal{R}$ represents all the system resources which are capacity controlled, i.e., it includes all the airports and air sectors which have greater projected demands than their capacities. The resources which are uncontrolled, do not have these imbalances in the demands and capacities, and are not included in $\mathcal{R}$ as they do not have any capacity constraints. The capacity of any controlled resource, $r \in R$ over a time interval t is denoted by b_{rt} and is specified in terms of the number of flights that the resource can hold during the interval t.

The set of all flight legs in the model is denoted by $\mathcal{F}$. For every flight leg $f \in \mathcal{F}$, the number of flight steps in the model are given by $|f|$. Each flight step in the original model corresponds to an entry into an air sector or an airport on a flight route. However, in our simplified model, since we neglect the impact of airspace congestion, we consider exactly two steps for each flight leg - the first step corresponding to takeoff from the origin airport and the second one corresponding to landing at the destination airport. Each flight step i of flight leg f is assigned an earliest start time $\alpha(f, i)$ and a duration time $\delta(f, i)$, both of which are determined from the original schedules.

The aircraft type for a flight f is an alphanumeric code designated to aircrafts by the International Civil Aviation Organization(ICAO) and is denoted in the model by $p(f)$. In addition, the origin and destination airports for flight f are denoted by $orig(f)$ and $dest(f)$ respectively. An aircraft routing $s \in \mathcal{S}$ represents the sequence $[f_1, f_2, ..., f_n]$ of flight legs carried out by an aircraft. The arrival and departure steps for two consecutive flight legs in an aircraft routing must have a minimum turn-around $m(p)$ between them (in the original model, the minimum turn-around m was assumed to be a constant instead of being a function of the aircraft type). The first and last flight legs for each routing $s \in \mathcal{S}$ are denoted by $[s]_1$ and $[s]_0$ respectively. In the model, the decision variables y_{fit} are binary and take 0-1 values, depending on whether the step i for flight f has started by time t or not. The table below summarizes the description of the model:

Indices and Sets

$t \in \mathcal{T}$	set of discrete time intervals,
$r \in \mathcal{R}$	set of capacity-controlled resources,
$f \in \mathcal{F}$	set of all flights,
$a \in \mathcal{A}$	set of all airports,
$p \in \mathcal{P}$	set of all aircraft types,
$s \in \mathcal{S}$	set of all aircraft routings,
$i \in \mathcal{I}(f)$	set of step indices in controlled flight plan for flight f

Data

$\lvert f \rvert$	number of steps in controlled flight plan for flight f,
$r(f,i)$	resource required by flight step i for flight f,
$\alpha(f,i)$	earliest start time for flight step i for flight f,
$\delta(f,i)$	processing time of flight step i for flight f,
$p(f)$	aircraft type for flight f,
$orig(f)$	origin airport for flight f,
$dest(f)$	destination airport for flight f,
$[\mathrm{s}]_0$	last flight in an aircraft routing s,
$[\mathrm{s}]_1$	first flight in an aircraft routing s,
$m(p)$	minimum turn-around time for an aircraft of type p.

Decision Variables

y_{fit}	1 if flight plan step i for flight f has started by time t and 0 otherwise,

Constraints

We next describe the constraints used in the model. Constraint 4.1 ensures that for flight leg $f \in \mathcal{F}$ and each flight step $i \in \mathcal{I}(f)$, the sequence of decision variables y_{fit} is monotonically increasing in t. This basically guarantees that if a flight step started by time t then it remains completed for all the future time periods.

$$y_{fit} \leq y_{fi(t+1)}, \quad \forall f \in \mathcal{F}, \forall i \in \mathcal{I}(f), \forall t \in \{0, ..., T-2\} \tag{4.1}$$

Constraint 4.2 requires that at the end of the time period, either the flight should have completed all the intermediate flights steps and reached its destination or it should have been cancelled and completed none of the flight steps.

$$y_{fi(T-1)} = y_{f(i+1)(T-1)} \quad \forall f \in \mathcal{F}, \forall i \in \mathcal{I}(f) \tag{4.2}$$

The next constraint 4.3 guarantees that a flight step cannot possibly cannot possibly start before its earliest start time. Specifically, for our simplified model, this directly translates into the condition that a flight cannot take-off or land before its original departure or arrival times respectively.

$$y_{fi(\alpha(f,i)-1)} = 0 \quad \forall f \in \mathcal{F}, \forall i \in \mathcal{I}(f) \text{ s.t. } \alpha(f,i) > 0 \tag{4.3}$$

The next constraint assumes that flight duration (i.e., time duration between the take-off step at the origin airport and the landing step at the destination airport) is a constant and equals $\alpha(f,2) - \alpha(f,1)$. This constraint basically ensures that no airborne delays are assigned in the optimized schedules.

$$y_{f(i+1)t} = y_{fi(t-\alpha(f,i+1)+\alpha(f,i))}, \quad \forall f \in \mathcal{F}, \forall i \in \mathcal{I}(f) \setminus \{|f|\} \tag{4.4}$$

The capacity constraints on controlled resources are enforced through Constraint 4.5. In this constraint, the term $\left(y_{fit} - y_{fi(t-\delta(f,i))}\right)$ denotes whether the controlled resource r is being used at time t for flight step i or not.

$$\sum_{\{(f,i):r(f,i)=\hat{r}\}} \left(y_{fit} - y_{fi(t-\delta(f,i))}\right) \leq b_{\hat{r}t}, \quad \forall \hat{r} \in \mathcal{R}, \forall t \in \mathcal{T} \tag{4.5}$$

Constraints 4.9 and 4.10 ensure that flow balance is maintained for all aircrafts of type $p \in P$ at each airport $a \in A$. However, before we look at those constraints in detail, we first define new variables $\Delta(a,p,t)$ which represent the difference between the number of aircrafts of type p arriving at an airport a by time $t - m(p)$ and the number of aircrafts of type p leaving the airport a by time t. We also note that if initially the number of aircrafts of type p at an airport a is zero, then $\Delta(a,p,t)$ represents the number of aircrafts of type p available for departure at time t from airport a.

$$\Delta(a,p,t) \triangleq \sum_{\{f\in\mathcal{F}:dest(f)=a,p(f)=p\}} y_{f|f|(t-m(p))} - \sum_{\{f\in\mathcal{F}:orig(f)=a,p(f)=p\}} y_{f1t} \tag{4.6}$$

For specifying the flow balance constraints, the model also needs to imconstraints which ensure that the number of aircrafts originating from and terminating at an airport (at the beginning and end of the day respectively) remains conserved. To achieve this objective, the model defines two constants, $\underline{n}(a,p,t)$ and $\overline{n}(a,p)$, both of which are calculated from the original flight schedules. While $\underline{n}(a,p,t)$ represents the count of routings s s of aircraft type p for which the first flights $[s]_1$ originate from airport a before time t, $\overline{n}(a,p)$ denotes the number of routings of type p aircrafts whose final flight legs $[s]_0$ have airport a as destination.

$$\underline{n}(a,p,t) \triangleq |\{s \in \mathcal{S} : orig([s]_1) = a, p([s]_1) = p, \alpha([s]_1, 1) \leq t\}| \tag{4.7}$$

$$\overline{n}(a,p) \triangleq |\{s \in S : dest([s]_0)) = a, p([s]_0) = p\}| \tag{4.8}$$

Using the variables and constants defined above, we can now specify the two flow balance constraints quite succinctly. The first flow balance constraint (constraint 4.9) ensures that for any airport a, the number of departures of type p aircrafts till time t cannot exceed the sum of the following two quantities: (1) the number of type p aircrafts arriving at airport a by time $(t-m(p))$, and (2) the number of type p aircrafts stationed at airport a since the beginning of the day and scheduled to begin their first flight legs from airport a before time t. The second constraint (constraint 4.10), on the other hand, ascertains that the end of day flow conservation requirements are satisfied. It states that the total number of type p aircrafts at the end of day at an airport a (denoted by $\overline{n}(a,p)$) must be equal to the sum of type p aircrafts initially stationed at airport a at the beginning of the day (denoted by $\underline{n}(a,p,T)$) and the net inflow of type p aircrafts into airport a during the day (denoted by $\Delta(a,p,T+m)$).

$$\Delta(a,p,t) + \underline{n}(a,p,t) \geq 0, \quad \forall a \in \mathcal{A}, \forall p \in \mathcal{P}, \forall t \in \mathcal{T} \tag{4.9}$$

$$\Delta(a,p,T+m) + \underline{n}(a,p,T) = \overline{n}(a,p), \quad \forall a \in \mathcal{A}, \forall p \in \mathcal{P} \tag{4.10}$$

Eq. 4.11 and 4.12 define the constraints on the cancellation and delay decision variables. Flight f is cancelled if $y_{f|f|(T-1)} = 0$, i.e., if by the end of the time period, the final flight step has not started. The delay $d(f)$ is the difference between the actual arrival time and scheduled arrival time of a flight. For all t greater than or equal to the actual arrival time $y_{f|f|t} = 1$. Thus, by subtracting off $y_{f|f|t}$ for those times from $T - \alpha(f,|f|)$, the delay is obtained. From constraint 4.12, we see that if a flight is cancelled, then the delay $d(f) = 0$. This is because for a cancelled flight $y_{f|f|t} = 0 \; \forall t$.

$$cx(f) = 1 - y_{f|f|(T-1)} \tag{4.11}$$

$$d(f) = (T - \alpha(f, |f|))y_{f|f|(T-1)} - \sum_{t=\alpha(f,|f|)}^{T-1} y_{f|f|t} \tag{4.12}$$

Objective function

As mentioned at the beginning of this section, the simplified version of the Airline Disruption Response model that we consider does not explicitly take into account the effect of passenger delay and missed connections costs. Thus, the objective function for our simplified model is made up of only two components: (1) estimated flight cancellation costs, and (2) superlinear flight operating costs associated with flight delays. This is in contrast with the original Airline Disruption Response model, in which the cancellation costs term was replaced by a cancellation benefits term for the airlines (for not having operate the cancelled flights) and cancellation costs were incorporated implicitly within an additional third term, which estimated passenger costs.

We now define the cancellation costs term

$$g^{canc}(\nu) = \nu \sum_{f \in \mathcal{F}} [ns(f)cx(f)] \tag{4.13}$$

Here, the estimated cancellation costs are controlled by the parameter ν, which determines the per seat cancellation cost taking into account the passenger costs associated with flight cancellations as well as the cancellation benefits of airlines for not having to operate the cancelled flight.

Fearing and Barnhart [22] argue that flight delays lead to operational costs due to several different reasons like missed connections, passenger delays, baggage delays, crew rescheduling, aircraft maintenance rescheduling, etc. Since in practice, only the airlines are privy to the internal costs associated with such operations, therefore, to estimate these costs, they use a superlinear cost function for approximation. The use of a superlinear function to model the costs is based on the notion that an increase in flight delays leads to a non-linear increase in operational costs. In the superlinear cost function defined in Eq. 4.14, the costs increase linearly with delay till the threshold of ψ is reached. Subsequent increases in delay above the threshold value results in a piecewise linear increase in costs, such that the slope of each piece is greater than its previous piece by a factor of λ. The parameter $\rho \in \mathbb{N}^+$ determines the length of each piece.

$$c_{ft} = min\{t - \alpha(f, |f|), \psi\} + \sum_{\epsilon=1}^{t-\alpha(f,|f|)-\psi} \lambda^{\text{ceil}(\frac{\epsilon}{\rho})} \tag{4.14}$$

Thus, using the superlinear delay cost function, the operational costs function $g^{ops}(\nu)$ is now defined in Eq. 4.15.

$$g^{ops}(\nu) = \zeta \sum_{f \in \mathcal{F}} \left[ns(f) \sum_{t=\alpha(f,|f|)}^{T-1} c_{ft}(y_{f|f|t} - y_{f|f|(t-1)}) \right] \tag{4.15}$$

To take into consideration the dependence of operational costs on the number of passengers on board on a flight, the parameter $0 < \zeta < 1$ is used to scale the seating capacities according to the expected demands. We note that for an operational flight $y_{f|f|t} - y_{f|f|(t-1)} = 1$ only for the time interval in which the final flight step $|f|$ is completed and thus, delay cost c_{ft} is imposed only for that step. We also see that if a flight f is cancelled, then $y_{f|f|t} = 0$ for all t and hence, no operational costs are incurred for cancelled flights.

4.2 Scenario Construction

For constructing the historical scenarios, we use an approach similar to the ones described in Barnhart et al. [7] and Fearing and Barnhart [22]. We first obtain the flight schedule data from the Flight Schedule Monitor(FSM), which is an ATFM decision support tool developed by Metron Aviation [26]. Among its several functionalities, the FSM provides data regarding flights' estimated arrival and departure times for all the transportation resources (airports and air sectors) that are encountered on the flight routes. However, since our simulations do not take the effect of air sectors into account, we consider the FSM data only to determine the departure and arrival times of flights at their origin and destination airports respectively. We obtain this data for a single day (April 23, 2007) of clear weather operations to estimate the planned flight schedules in the Official Airline Guide. As described in Fearing and Barnhart [22], we then subset the data thus obtained by considering only the 20 airlines (refer to Table 4.1) which are represented in the Airline Service Quality Performance (ASQP) 2007 dataset and also restricting the origin-destination flight routes to include only those that are in the T-100 Domestic Segment database. This reduces the number of flights from around 38,000 in our original dataset to around 16000 in the subset dataset. Metron Aviation's FSM data also provides information about the GDP programs which have been implemented previously. This data contains all the relevant details about the GDP program like the program resource, reporting and im-

Airline Codes	Airline Names
9E	Pinnacle Airlines
AA	American Airlines
AQ	Aloha Airlines
AS	Alaska Airlines
B6	JetBlue Airways
CO	Continental Airlines
DL	Delta Airlines
EV	Atlantic Southeast Airlines
F9	Frontier Airlines
FL	AirTran Airways
HA	Hawaiin Airlines
MQ	American Eagle Airlines
NW	Northwest Airlines
OH	Comair
OO	SkyWest Airlines
UA	United Airlines
US	US Airways
WN	Southwest Airlines
XE	ExpressJet Airlines
YV	Mesa Airlines

Table 4.1: IATA codes for the airlines considered in simulations

plementation time, duration and capacity for each 15 minute interval. For performing our simulations, we consider the FSM data for the same set of 10 representative TFM scenarios, which were considered by Barnhart et al. [7] and Fearing and Barnhart [22]. However, since we want to study the impact of prioritization at individual airports, we further break each TFM scenario on the basis of the GDP resources. The details of the 40 GDP scenarios thus obtained are given in Appendix A. The corresponding information regarding the airports considered in these scenarios are provided in Table 4.2.

Airport Code	Airport Name
ATL	HartsfieldJackson Atlanta International Airport
DCA	Ronald Reagan Washington National Airport
EWR	Newark Liberty International Airport
IAD	Washington Dulles International Airport
JFK	John F. Kennedy International Airport
LGA	LaGuardia Airport
MDW	Chicago Midway International Airport
ORD	Chicago O'Hare International Airport
PHL	Philadelphia International Airport
SFO	San Francisco International Airport

Table 4.2: Airports considered for constructing simulation scenarios

4.3 Generating Aircraft Routings

For tracking an aircraft's routings, we need information about its tail number. The flights in the FSM data, however, do not have tail numbers of aircrafts associated with them, and while it is possible to match the tail numbers of aircrafts using the ASQP database, it is sometimes found that the tail numbers provided in the ASQP database are incorrect. This results in the generation of aircraft routings, which are invalid or infeasible. Fearing and Barnhart [22] define a valid or feasible routing s to be one for which the following conditions hold true:

1. The origin airport for a flight leg is the same as the destination airport of the previous flight leg in the routing, i.e., $orig([s]_{k+1}) = dest([s]_k)\ \forall k$.
2. The aircraft type remains the same for all flights in the routing, i.e., $p([s]_1) = p([s]_2) = ... = p([s]_0)$.
3. The minimum turnaround time $m(p([s]_0))$ is maintained between any two flight

legs, i.e., $\alpha([s]_{k+1}, 1) \geq \alpha([s]_k, |[s]_k|) + m(p([s]_0))\ \forall k$.

For generating valid aircraft routings, we use the procedure described by Fearing and Barnhart [22]. We first generate the planned aircraft routings, based on the tail numbers in ASQP database. In case of discrepancies, we select the longest sub-sequence compatible with our validity rules and greedily assign the inconsistent flights to the aircraft routings which are compatible. Following this method, we get in all 4451 aircraft routings for the 16,129 FSM flights scheduled in the 24-hour time period.

4.4 Airline Bidding Strategies

For performing our simulations, we consider two different bidding strategies for the airlines. In the first strategy, we assume the airline bids at an airport to be proportional to the total number of seats on all of their respective incoming flights at that airport, i.e., we assume

$$b_{ra} \propto \sum_{\{f \in \mathcal{F}_a : dest(f) = r\}} ns(f) \tag{4.16}$$

In the above equation, $\mathcal{F}_a$ denotes the set of all airline a flights. Using this strategy, we obtain the bids as shown in Table 4.3. From the table, we observe that for air carriers operating on a hub-and-spoke model, the strategy of allocating bids proportionally to the number of seats on all of their incoming flights at an airport, corresponds to a great extent to the strategy of prioritizing their hub airports. For example, in Table 4.3 we see that both Delta (IATA Code: DL) and United (IATA Code: UA) Airlines have a significant portion of their bid budgets allocated to their respective

AIRPORTS	AIRLINES																			
	9E	**AA**	**AQ**	**AS**	**B6**	**CO**	**DL**	**EV**	**F9**	**FL**	**HA**	**MQ**	**NW**	**OH**	**OO**	**UA**	**US**	**WN**	**XE**	**YV**
ATL	0	74	0	0	0	37	719	590	31	446	0	13	107	42	164	13	50	0	47	35
DCA	156	99	0	49	0	37	55	7	31	10	0	71	159	62	0	28	28	0	30	0
EWR	0	58	0	34	36	523	22	9	0	7	0	13	94	12	0	24	40	0	738	11
IAD	117	32	0	0	37	0	15	0	0	16	0	12	41	30	0	169	9	111	65	302
JFK	0	114	0	0	305	11	82	0	0	0	0	55	42	204	0	27	43	0	18	28
LGA	117	215	0	0	21	38	114	2	21	25	0	131	162	132	0	47	0	0	17	23
MDW	0	0	0	0	0	3	0	9	42	51	0	0	58	9	0	0	0	1968	12	0
ORD	0	675	0	43	16	43	17	3	0	0	0	901	139	40	699	474	93	0	30	253
PHL	195	53	0	0	0	28	20	3	10	38	0	0	84	12	11	29	60	586	15	0
SFO	0	139	0	196	0	48	40	2	71	5	107	17	96	0	412	313	95	0	0	4
Total	**585**	**1459**	**0**	**322**	**415**	**768**	**1084**	**625**	**206**	**598**	**107**	**1213**	**982**	**543**	**1286**	**1124**	**418**	**2665**	**972**	**656**

Table 4.3: Airline bids based on the total number of seats on all the incoming flights

hubs in Atlanta (IATA Code: ATL) and Chicago (IATA Code: ORD). We have already discussed this idea in considerable detail through our hypothetical example in Section 2.3 and shown that significant cost benefits can be achieved using this strategy for both passengers as well as airlines.

The second strategy that we use is based on the concept of delay multipliers developed by Beatty et al. [21]. The delay multiplier is a metric used to evaluate the propagation of an initial flight delay from a bottleneck airport to other airports in the network due to connection delays. The delays in flight connections could be due to different reasons like aircraft delays, passenger delays, crew delays or baggage delays. Due to the unavailability of data, however, in our simulations, we consider only the effect of the late arrival of an aircraft for calculating the delay multipliers. Beatty et al. [21] define the delay multiplier DM for a flight as :

$$DM = \frac{I + D}{I} \tag{4.17}$$

where I= Initial delay and D=Down line delay (both in minutes). For clarity, consider the following example. Suppose that at JFK, an aircraft gets delayed by 1 hour. Because of this initial delay, assume that the next 2 flight legs for the aircraft get delayed by 45 and 15 minutes respectively. Then, according to the above definition, $I = 60$ min, $D = 45 + 15 = 60$ min and the delay multiplier, $DM = 2$. Using this definition, the authors in their paper had studied the temporal variation of delay multipliers during the day based on actual American Airlines crew and aircraft schedules.

We see from Eq. 4.17 that DM is a measure of delay propagation for an individual flight. However, in order to employ this concept to determine the strategies for airline bids at each airport, we need a metric that provides the airlines a measure of delay propagation for all of their flights at each airport. In addition, we also need this metric to take into account the fact that airlines have a flexibility to optimize their schedules within the RBS allocated slots. We therefore, use the Airline Disruption Response model and combine it with a Monte Carlo simulation approach to redefine the delay multiplier $\lambda^{\omega}_{r(\omega)a}$ as follows:

$$\lambda^{\omega}_{r(\omega)a} = \frac{\sum\limits_{f \in \mathcal{F}_a} (d^{*\omega}(f) + \gamma \ cx^{*\omega}(f))}{\sum\limits_{\{f \in \mathcal{F}_a : dest(f) = r(\omega)\}} (RBS^{\omega}(f) - \alpha(f, |f|))} \tag{4.18}$$

In the above equation, $\lambda^{\omega}_{r(\omega)a}$ denotes the delay multiplier for airline a at controlled airport $r(\omega)$ under scenario ω. $d^{*\omega}(f)$ and $cx^{*\omega}(f)$ respectively represent the optimal values of decision variables $d(f)$ and $cx(f)$ obtained from the Airline Disruption Response model for scenario ω. $RBS^{\omega}(f)$ is the schedule obtained by applying RBS to the original schedule at controlled airport $r(\omega)$ under scenario ω. γ denotes the penalty parameter associated with the cancellation of flight f. In our experiments, we take $\gamma = 6$ hours, i.e., we assume the delay associated with a cancelled flight to be equivalent to 6 hours. We can thus observe from Eq. 4.18 that $\lambda^{\omega}_{r(\omega)a}$ is the ratio of the total propagated delays (over the airports network) to the total incoming delays for an airline a at a reduced capacity airport r under the GDP scenario ω. Since the propagated delay is atleast as large as the initial delay, therefore, $\lambda^{\omega}_{r(\omega)a}$ is always greater than or equal to 1.

We now define the parameter λ_{ra} in terms of $\lambda^{\omega}_{r(\omega)a}$ as follows:

$$\lambda_{ra} = \frac{\sum\limits_{\{\omega\in\Omega:\ r(\omega)=r\}} \lambda^{\omega}_{r(\omega)a} N^{\omega}_{r(\omega)a}}{\sum\limits_{\{\omega\in\Omega:\ r(\omega)=r\}} N^{\omega}_{r(\omega)a}} \tag{4.19}$$

In the above equation, N^{ω}_{ra} denotes the number of flights of airline a affected at airport r during GDP scenario ω. Since, λ_{ra} is a weighted average of the $\lambda^{\omega}_{r(\omega)a}$, which are all greater than or equal to 1. Therefore, the values of λ_{ra} are also greater than 1. The values of λ_{ra}, obtained through our simulations on the 40 representative scenarios are summarised in Table 4.4. We see that for many of the airlines and airports, the delay multipliers are undefined. This can be explained on the basis of equations 4.18 and 4.19. Whenever the RBS allocated schedule is the same as the original airline schedule, the initially allocated delay itself is zero and hence, no delay propagation occurs over the network. Therefore, the delay multiplier λ_{ra} is undefined. Similarly, from Eq. 4.19, we see that if no flights are scheduled for an airlines (more specifically, during the duration of the GDP Scenario), then none of its flights will have to be rescheduled. In this case again, the initially allocated delay is 0 and the delay multiplier is undefined. We also observe from Table 4.4 that some of the delay multipliers have very large values. This is due to the fact that we impose a large penalty of γ on flight cancellations.

For determining the bids for airlines based on the delay multipliers approach, we first note that larger values of delay multipliers correspond to larger delay propagation over the network. Therefore, in order to reduce delays, we assume that the airlines set their bids b_{ra} to be proportional to the delay multipliers λ_{ra}. The bids

AIRPORTS	AIRLINES																			
	9E	**AA**	**AQ**	**AS**	**B6**	**CO**	**DL**	**EV**	**F9**	**FL**	**HA**	**MQ**	**NW**	**OH**	**OO**	**UA**	**US**	**WN**	**XE**	**YV**
ATL	-	1.3	-	-	-	1.33	1	1	-	1	-	2	1.32	1.1	1.57	1	2	-	1	9
DCA	-	-	-	-	-	2	1	1	1	1	-	2	2.33	2	-	1	2	-	-	-
EWR	-	1.24	-	1	1	1.03	1	1	-	1	-	1.28	1.7	1.4	-	1	1.14	-	1	-
IAD	-	2.44	-	-	1	-	1	-	-	-	-	2	1.95	1.1	-	1	-	1	1	1.26
JFK	-	2.43	-	-	1	-	1	-	-	-	-	1.4	4.33	1.41	-	1	1.67	-	-	4.04
LGA	-	1.51	-	-	1	1.68	1	-	1	1	-	1.19	2.49	1.48	-	1	-	-	1	3.67
MDW	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-
ORD	-	1.01	-	1	1	1.09	1	1	-	-	-	1	1.21	1.09	1.07	1	1.11	-	1	1.13
PHL	-	-	-	-	-	-	1	-	-	-	-	-	-	-	-	1	-	1	-	-
SFO	-	1.63	-	1	-	2.47	-	-	1	-	-	-	5	-	1.69	1	2	-	-	-

Table 4.4: Delay Multipliers for all the airlines at all the airports

AIRLINES	**AIRLINES**																			
	9E	**AA**	**AQ**	**AS**	**B6**	**CO**	**DL**	**EV**	**F9**	**FL**	**HA**	**MQ**	**NW**	**OH**	**OO**	**UA**	**US**	**WN**	**XE**	**YV**
ATL	0	20	0	0	0	13	296	240	3	185	0	2	14	9	19	6	5	0	8	7
DCA	4	27	0	3	0	12	27	3	3	4	0	21	17	14	0	13	3	0	5	0
EWR	0	15	0	2	12	143	9	3	0	3	0	4	13	2	0	10	4	0	125	3
IAD	3	8	0	0	15	0	7	0	0	7	0	3	5	7	0	63	1	10	11	78
JFK	0	23	0	0	114	4	33	0	0	0	0	15	5	45	0	11	4	0	3	5
LGA	3	57	0	0	7	12	53	1	2	10	0	39	17	30	0	18	0	0	3	6
MDW	0	0	0	0	0	1	0	3	4	21	0	0	8	2	0	0	0	179	2	0
ORD	0	179	0	3	6	14	8	1	0	0	0	198	18	8	106	193	8	0	5	62
PHL	5	14	0	0	0	8	9	1	1	15	0	0	9	2	2	13	6	53	3	0
SFO	0	32	0	14	0	12	13	1	7	2	1	5	9	0	90	116	9	0	0	1
Total	**15**	**375**	**0**	**22**	**154**	**219**	**455**	**253**	**20**	**247**	**1**	**287**	**115**	**119**	**217**	**443**	**40**	**242**	**165**	**162**

Table 4.5: Numbers of flights scheduled by airlines at disurption airports

AIRPORTS	**AIRLINES**																			
	9E	**AA**	**AQ**	**AS**	**B6**	**CO**	**DL**	**EV**	**F9**	**FL**	**HA**	**MQ**	**NW**	**OH**	**OO**	**UA**	**US**	**WN**	**XE**	**YV**
ATL	0	164	0	0	0	106	135	157	0	150	0	223	64	62	466	125	85	0	194	310
DCA	146	0	0	0	0	160	135	157	69	150	0	223	112	113	0	125	85	0	0	0
EWR	0	156	0	108	83	83	135	157	0	150	0	143	82	79	0	125	48	0	194	0
IAD	146	308	0	0	83	0	135	0	0	0	0	223	94	62	0	125	0	888	194	43
JFK	0	307	0	0	83	0	135	0	0	0	0	156	209	80	0	125	70	0	0	139
LGA	146	191	0	0	83	134	135	0	69	150	0	133	120	84	0	125	0	0	194	126
MDW	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	888	0	0
ORD	0	127	0	108	83	87	135	157	0	0	0	112	58	62	318	125	47	0	194	39
PHL	146	0	0	0	0	0	135	0	0	0	0	0	0	0	0	125	0	888	0	0
SFO	0	206	0	108	0	197	0	0	69	0	107	0	241	0	503	125	85	0	0	0
Total	**584**	**1459**	**0**	**324**	**415**	**767**	**1080**	**628**	**207**	**600**	**107**	**1213**	**980**	**542**	**1287**	**1125**	**420**	**2664**	**970**	**657**

Table 4.6: Airline bids based on the delay multipliers calculated in Table 4.4

obtained by implementing this procedure are given in Table 4.6. While evaluating the bids for airlines which have undefined delay multipliers for all the scenarios (for example, airlines 9E and HA), we place zero bids at those airports where they have no flights scheduled (refer to Table 4.5), but split the bids equally among all the other airports where they have flights scheduled. This is because even though flights may not have initial delays due to disruptions, after prioritization these airlines may incur delays because of the priorities which other airlines gain. For those airlines, which have zero flights scheduled at all the disruption airports (for example, airline AQ), we simply set all the bids to be zero.

Based on their bids, the airlines receive priorities according to the expression given in Eq. 3.1. As we had discussed in section 1.2, P_r, the pool of prioritization points assigned to airport r, provides regulator the ability to control the level of prioritization at different airports. In order to see the effect of varying P_r, we consider two different values for it. In the first case, we assume $P_r = \sum_a N_{ra}$, while in the second we set $P_r = 2\sum_a N_{ra}$. The rationale behind expressing P_r as a multiple of N_{ra} is that we assume that the regulator assigns some priority points (in our experiments, 1 and 2 points respectively) for each flight scheduled at an airport, which depending on the bids get divided differently among the different airlines. Based on the two strategies used for bid allocation (one using the number of seats as a criterion and the using second delay multipliers), the number of priority minutes assigned are given in Tables 4.7 - 4.10. Comparing the two strategies based on the number of priority minutes in these tables, we see that when number of seats is used as a criterion, the priorities assigned are relatively smaller but are allocated at more airports. This is because under the number of seats criterion, the airlines bid at all the airports where their flights are scheduled. On the other hand, the delay multipliers criterion favours higher bidding for some airports (where multipliers are larger). We note here that both the strategies have their own advantages. While bidding higher at fewer airports provides airlines more chances of gaining priorities at those airports, the strategy of placing smaller bids at a larger number of airports prevents airlines from losing out priorities to other airlines.

AIRPORTS	**AIRLINES**																			
	9E	**AA**	**AQ**	**AS**	**B6**	**CO**	**DL**	**EV**	**F9**	**FL**	**HA**	**MQ**	**NW**	**OH**	**OO**	**UA**	**US**	**WN**	**XE**	**YV**
ATL	0	1.3	0	0	0	1	0.8	0.9	3.6	0.8	0	2.2	2.7	1.6	3	0.8	3.5	0	2	1.8
DCA	7.4	0.7	0	3.1	0	0.6	0.4	0.4	2	0.5	0	0.6	1.8	0.8	0	0.4	1.8	0	1.1	0
EWR	0	0.8	0	3.6	0.6	0.8	0.5	0.7	0	0.5	0	0.7	1.5	1.3	0	0.5	2.2	0	1.3	0.8
IAD	8.9	0.9	0	0	0.6	0	0.5	0	0	0.5	0	0.9	1.9	1	0	0.6	2.1	2.5	1.3	0.9
JFK	0	1.4	0	0	0.8	0.8	0.7	0	0	0	0	1	2.3	1.3	0	0.7	3	0	1.7	1.6
LGA	9.4	0.9	0	0	0.7	0.8	0.5	0.5	2.5	0.6	0	0.8	2.3	1.1	0	0.6	0	0	1.3	0.9
MDW	0	0	0	0	0	0.3	0	0.3	1.1	0.2	0	0	0.7	0.4	0	0	0	1.1	0.6	0
ORD	0	0.9	0	3.4	0.6	0.7	0.5	0.7	0	0	0	1.1	1.8	1.2	1.6	0.6	2.8	0	1.4	1
PHL	4.8	0.5	0	0	0	0.4	0.3	0.4	1.3	0.3	0	0	1.1	0.7	0.7	0.3	1.2	1.4	0.6	0
SFO	0	0.9	0	2.8	0	0.8	0.6	0.4	2.1	0.5	21.6	0.7	2.2	0	0.9	0.5	2.1	0	0	0.7

Table 4.7: Priority minutes based on number of seats assigned to each flight when $P_r = \sum_a N_{ra}$.

AIRPORTS	AIRLINES																			
	9E	**AA**	**AQ**	**AS**	**B6**	**CO**	**DL**	**EV**	**F9**	**FL**	**HA**	**MQ**	**NW**	**OH**	**OO**	**UA**	**US**	**WN**	**XE**	**YV**
ATL	0	2.6	0	0	0	2	1.7	1.7	7.3	1.7	0	4.4	5.3	3.3	6	1.6	6.9	0	4.1	3.5
DCA	14.8	1.4	0	6.2	0	1.2	0.8	0.8	4	1	0	1.3	3.5	1.7	0	0.8	3.6	0	2.3	0
EWR	0	1.7	0	7.3	1.3	1.6	1.1	1.3	0	1	0	1.4	3.1	2.6	0	1	4.3	0	2.5	1.6
IAD	17.8	1.8	0	0	1.1	0	1	0	0	1	0	1.9	3.8	2	0	1.2	4.3	5	2.7	1.8
JFK	0	2.8	0	0	1.5	1.6	1.4	0	0	0	0	2.1	4.7	2.6	0	1.4	6.1	0	3.4	3.2
LGA	18.9	1.8	0	0	1.5	1.5	1	1.1	5.1	1.2	0	1.6	4.6	2.1	0	1.3	0	0	2.7	1.9
MDW	0	0	0	0	0	0.6	0	0.6	2.1	0.5	0	0	1.5	0.9	0	0	0	2.2	1.2	0
ORD	0	1.8	0	6.8	1.3	1.5	1	1.5	0	0	0	2.1	3.6	2.4	3.1	1.2	5.5	0	2.8	1.9
PHL	9.6	0.9	0	0	0	0.9	0.6	0.8	2.6	0.6	0	0	2.3	1.5	1.4	0.5	2.5	2.7	1.2	0
SFO	0	1.8	0	5.7	0	1.6	1.2	0.9	4.1	1.1	43.2	1.4	4.3	0	1.8	1.1	4.3	0	0	1.5

Table 4.8: Priority minutes based on number of seats assigned to each flight when $P_r = 2\sum_a N_{ra}$.

AIRPORTS	**AIRLINES**																			
	9E	**AA**	**AQ**	**AS**	**B6**	**CO**	**DL**	**EV**	**F9**	**FL**	**HA**	**MQ**	**NW**	**OH**	**OO**	**UA**	**US**	**WN**	**XE**	**YV**
ATL	0	3	0	0	0	3	0.2	0.2	0	0.3	0	41.2	1.7	2.6	9.1	7.7	6.2	0	9	16.3
DCA	3.9	0	0	0	0	1.4	0.5	5.5	2.4	4	0	1.1	0.7	0.9	0	1	3	0	0	0
EWR	0	2.4	0	12.2	1.6	0.1	3.4	11.8	0	11.2	0	8	1.4	8.9	0	2.8	2.7	0	0.4	0
IAD	6.2	4.9	0	0	0.7	0	2.4	0	0	0	0	9.4	2.4	1.1	0	0.3	0	3.9	2.2	0.1
JFK	0	2.7	0	0	0.1	0	0.8	0	0	0	0	2.1	8.4	0.4	0	2.3	3.5	0	0	5.6
LGA	7.4	0.5	0	0	1.8	1.7	0.4	0	5.3	2.3	0	0.5	1.1	0.4	0	1.1	0	0	9.9	3.2
MDW	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.2	0	0
ORD	0	0.3	0	17.6	6.8	3	8.3	76.6	0	0	0	0.3	1.6	3.8	1.5	0.3	2.9	0	19	0.3
PHL	2.1	0	0	0	0	0	1.1	0	0	0	0	0	0	0	0	0.7	0	2.1	0	0
SFO	0	1.2	0	1.5	0	3.1	0	0	1.9	0	20.4	0	5.1	0	1.1	0.2	1.8	0	0	0

Table 4.9: Priority minutes based on delay multipliers assigned to each flight when $P_r = \sum_a N_{ra}$.

AIRPORTS	AIRLINES																			
	9E	**AA**	**AQ**	**AS**	**B6**	**CO**	**DL**	**EV**	**F9**	**FL**	**HA**	**MQ**	**NW**	**OH**	**OO**	**UA**	**US**	**WN**	**XE**	**YV**
ATL	0	6.1	0	0	0	6	0.3	0.5	0	0.6	0	82.3	3.4	5.1	18.1	15.4	12.5	0	17.9	32.7
DCA	7.7	0	0	0	0	2.8	1.1	11	4.9	7.9	0	2.2	1.4	1.7	0	2	6	0	0	0
EWR	0	4.7	0	24.3	3.1	0.3	6.8	23.6	0	22.5	0	16.1	2.8	17.9	0	5.6	5.4	0	0.7	0
IAD	12.3	9.7	0	0	1.4	0	4.9	0	0	0	0	18.8	4.8	2.3	0	0.5	0	7.8	4.5	0.1
JFK	0	5.4	0	0	0.3	0	1.6	0	0	0	0	4.2	16.8	0.7	0	4.6	7.1	0	0	11.2
LGA	14.9	1	0	0	3.6	3.4	0.8	0	10.5	4.6	0	1	2.2	0.9	0	2.1	0	0	19.8	6.4
MDW	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2.5	0	0
ORD	0	0.7	0	35.1	13.6	6.1	16.6	153.3	0	0	0	0.6	3.2	7.6	2.9	0.6	5.8	0	38	0.6
PHL	4.2	0	0	0	0	0	2.2	0	0	0	0	0	0	0	0	1.4	0	4.2	0	0
SFO	0	2.5	0	2.9	0	6.2	0	0	3.8	0	40.7	0	10.2	0	2.1	0.4	3.6	0	0	0

Table 4.10: Priority minutes based on delay multipliers assigned to each flight when $P_r = 2\sum_a N_{ra}$.

Chapter 5

Results and Conclusions

In the previous chapter, we had completed our discussion regarding the framework used for carrying out our experiments and additionally, described two different strategies the airlines can use for determining their bids. We also saw that depending on the pool of priority points set by the regulator, the airlines received different priority allocations. In this chapter, we analyze the results that we obtain from these prioritized schedules and draw conclusions drawn from the thesis.

5.1 Simulation Results

In our experiments, based on the priority minutes given in Tables 4.7 - 4.10, we generate prioritized schedules by subtracting off the number of priority minutes allocated for each flight, as described in 2.2. For rationing slots at reduced capacity airports, however, instead of using the Modified RBPS algorithm, we directly apply RBS on the prioritized schedules and reschedule the flights at their original scheduled times if the allocated slots obtained by this procedure are found to be earlier than the original scheduled times.

We now discuss the parameters used in the Airline Disruption Response Model to determine the airlines' optimal slots for their flights. As discussed in Section 4.1, the objective function is defined by five parameters: ν (the penalty cost per seat for a cancelled flights), ψ (the operating cost threshold), ρ (length of each piece in the super-linear operating cost), λ (the growth factor for the slope of each piece in the operating cost) and ζ (the scaling factor for the number of seats based on expected demands). Based on the *aggressive approach* described in Fearing and Barnhart [22], we set the values of the parameters to be ζ=0.5, ψ=6, λ=1.5 and ρ=3. However, since ν in our model represents the penalty cost, therefore we set ν=54. This is based on the assumption that the delay for a passenger due to cancelled flight is around 6 hours (=72 five-minute intervals) and the load factor of flights is between 0.8-0.85. Taking this into account and assuming a cost benefit of 6 units (as explained in Fearing and Barnhart [22]), we get $\nu \approx 54$.

Once the slots have been rationed based on the reduced capacities of the airports, we use the Airline Disruption Response Model to evaluate the costs of airlines over all the 40 scenarios. Table 5.1 shows the results that we get from optimization. In this table, the No Prioritization column corresponds to the Total Costs obtained by simply applying RBS on the Original Schedule (without any prioritization) and optimizing the schedules using the Airline Disruption Response Model. The next four columns show the optimized costs for schedules prioritized on the basis of number of seats and delay multipliers respectively. We first note from these columns that for both the prioritization schemes, the total costs for each airline over all the 40 scenarios is less than or equal to the their original costs (given in No Prioritization column). This clearly shows that by prioritizing their flights, none of the airlines are made worse-off. In fact, as can be seen from Percentage Reduction in Costs column, nearly all of the airlines are strictly better off. From a viewpoint of the regulator, we see that by increasing the pool of points P_r from 1 min/scheduled flight to 2 min/per scheduled flight at airport r, the prioritized costs decrease even further for both the strategies. Considering the combined costs of all the airlines together, we see that prioritizing flights by the number of seats results in 10.4% and 17.6% decreases in costs for $P_r = 1$ min/flight and $P_r = 2$ min/flight respectively. The corresponding numbers for the strategy involving delay multipliers are 7.6% for $P_r = 1$ min/flight and 13.6% for $P_r = 2$ min/flight.

From the simulations, we also note that the prioritization strategy based on the number of seats consistently does better than the one based on delay multipliers. Based on our discussions in the last chapter, this suggests that allocating relatively lower bids at a larger number of airports works better than allocating very high bids at only a few airports. In order to study the effect of airline bids on its internal costs, we plot figures 5.1-5.4, which show the scatter plots of the percentage change in original costs with the per flight airline bids at each airport for the two strategies at the two different P_r values. Each point on the scatterplot corresponds to the percentage change in the cost to an airline based on its per flight bid at some airport. We see from all the the plots there appears to be a strong positive correlation (varying from 0.4 to 0.7) between the percentage decrease in internal costs and the per flight bids of an airline at an airport. We note it is the per flight bid (not just the overall bid) of an airline at an airport, which is the more relevant factor in influencing flight costs. This is quite intuitive since the priority minutes for an airline are equally split among its flights at all the airports.

AIRLINES	TOTAL COSTS FOR ALL 40 SCENARIOS					% REDUCTION IN COSTS OVER ALL 40 SCENARIOS			
	No Prioritization	Prioritization Based on Number of Seats		Prioritization Based on Delay Multipliers		Prioritization Based on Number of Seats		Prioritization on Delay Multipliers	
		$P_r = \sum_a N_{ra}$	$P_r = 2\sum_a N_{ra}$	$P_r = \sum_a N_{ra}$	$P_r = 2\sum_a N_{ra}$	$P_r = \sum_a N_{ra}$	$P_r = 2\sum_a N_{ra}$	$P_r = \sum_a N_{ra}$	$P_r = 2\sum_a N_{ra}$
9E	0	0	0	0	0	-	-	-	-
AA	55695	49848	45411	51206	47755	10.5 %	18.5 %	8.1 %	14.3 %
AQ	0	0	0	0	0	-	-	-	-
AS	5500	4378	3476	2876	1616	20.4 %	36.8 %	47.7 %	70.6 %
B6	23175	19750	17625	22475	21500	14.8 %	23.9 %	3 %	7.2 %
CO	84603	77377	72781	84063	80917	8.5 %	14 %	0.6 %	4.4 %
DL	49445	44778	41524	44618	41855	9.4 %	16 %	9.8 %	15.4 %
EV	9362	8802	8372	8513	7912	6 %	10.6 %	9.1 %	15.5 %
F9	199	133	0	133	66	33.2 %	100 %	33.2 %	66.8 %
FL	19700	19037	17944	18034	17028	3.4 %	8.9 %	8.5 %	13.6 %
HA	0	0	0	0	0	-	-	-	-
MQ	12334	11331	10238	11513	10303	8.1 %	17 %	6.7 %	16.5 %
NW	15794	13369	11680	14074	13064	15.4 %	26 %	10.9 %	17.3 %
OH	4225	3330	3010	3425	2610	21.2 %	28.8 %	18.9 %	38.2 %
OO	8919	7976	7156	7523	6079	10.6 %	19.8 %	15.7 %	31.8 %
UA	51605	45960	42707	46862	43280	10.9 %	17.2 %	9.2 %	16.1 %
US	6170	5212	4401	5099	4525	15.5 %	28.7 %	17.4 %	26.7 %
WN	959	548	411	411	274	42.9 %	57.1 %	57.1 %	71.4 %
XE	22838	20500	18865	22204	21960	10.2 %	17.4 %	2.8 %	3.8 %
YV	7338	6280	5683	6281	5800	14.4 %	22.6 %	14.4 %	21 %
Total	**377864**	**338611**	**311286**	**349312**	**326546**	**10.4 %**	**17.6 %**	**7.6 %**	**13.6 %**

Table 5.1: Comparison of Prioritization Strategies based on Total Costs and Percentage Reduction in Costs for $P_r = \sum_a N_{ra}$ and $P_r = 2\sum_a N_{ra}$

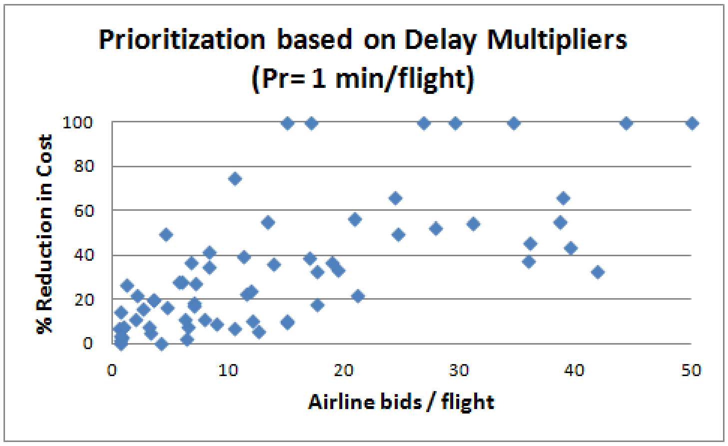

Figure 5.1: Scatter Plot of Percentage Reduction in Cost v/s Airline Bids per Flight for $P_r = \sum_a N_{ra}$ (=1 flight/min)

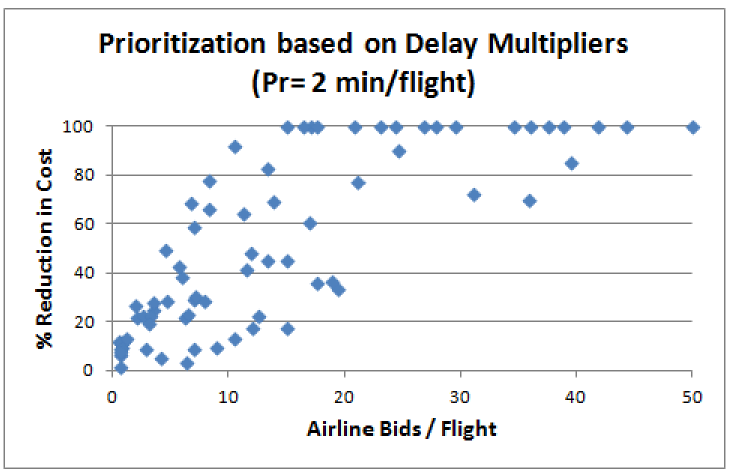

Figure 5.2: Scatter Plot of Percentage Reduction in Cost v/s Airline Bids per Flight for $P_r = 2\sum_a N_{ra}$ (=2flights/min)

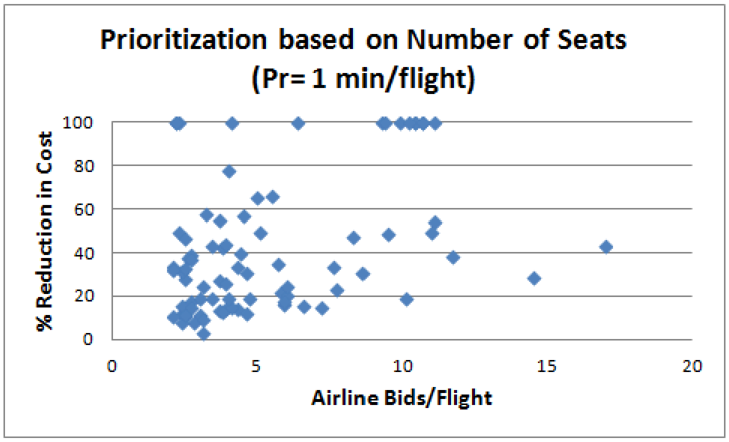

Figure 5.3: Scatter Plot of Percentage Reduction in Cost v/s Airline Bids per Flight for $P_r = \sum_a N_{ra}$ (=1 flight/min)

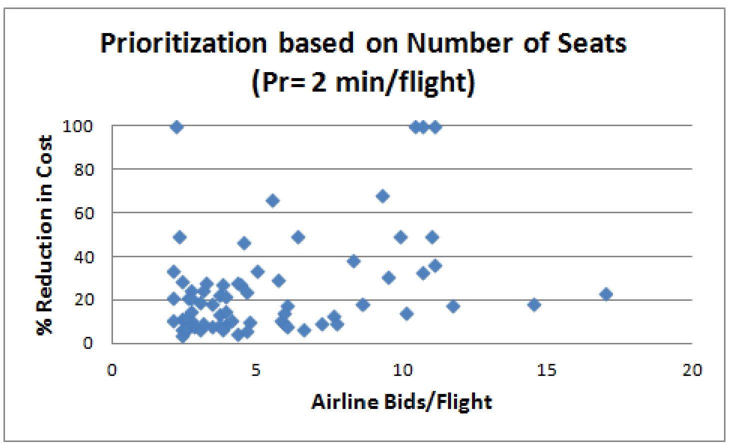

Figure 5.4: Scatter Plot of Percentage Reduction in Cost v/s Airline Bids per Flight for $P_r = 2\sum_a N_{ra}$ (=2flights/min)

5.2 Conclusions and Scope for Future Study

In this thesis, we explored and developed in detail the strategic prioritization framework proposed by Fearing and Kash [1] for airline scheduling during disruptions. Due to fairness considerations, the airline rescheduling procedure currently implemented by the FAA does not take into consideration the individual preferences of the airlines while allocating the initial schedules. This causes several airline operations to be adversely affected whenever disruptions occur at airports due to bad weather conditions or severe air traffic congestion.

The strategic prioritization framework, which is a simple extension of the current system in operation, provides the airlines opportunities to prioritize the airports according to their needs. The allocation of priorities in this framework takes place through a two stage non-monetary game in which the airlines bid for priorities using points allocated to them by the regulator. In this thesis, we first developed motivation about why such a framework could work when implemented in practice. We then extended the theoretical results for this framework to show that that it possesses several desirable properties in the second stage of the game, like the existence of a Nash equilibrium for the general multiple airlines and multiple airports case, and the uniqueness of the equilibrium for the two airlines and multiple airports case.

A major contribution of the thesis was that we were able to show through computer simulations on historical flight data that strategic prioritization yielded lower operational costs for all the airlines. For our analysis, we considered two different bidding strategies for the airlines. The first strategy was based on simply setting the priorities proportional to the number of total seats on all incoming flights at an airport. The second strategy involved extending the concept of delay multipliers developed by Beatty et al. [21] using the Airline Disruption Response model based on Monte Carlo simulations. Using these two different bidding strategies, significant reductions (upto 17.6%) were achieved in the total operational cost of the airlines.

The strategic prioritization approach showed promising results through simulations on our representative historical scenarios. However, a key limitation in our approach was that we did not implement the Modified RBPS algorithm that we developed in Section 2.2 for our simulations. This potentially led to some of the capacity constraints being violated for the reduced capacity airport. It would be interesting to analyze the effect which the implementation of Modified RBPS algorithm has on the operational costs of the airlines. Another limitation in our approach was that the scenarios that we used to simulate the impact of priority minutes consisted of only one controlled airport. While for determining the delay multipliers, the approach of using a single controlled airport per scenario is probably the best, for studying the effect of prioritization overall, the multiple resource scenarios as constructed in Fearing and Barnhart [22] might be more useful to analyze.

On the theoretical front, we would like to derive conditions under which a unique pure strategy Nash equilibrium exists. As discussed in 3.2.2, this would involve proving that the utility functions of the airlines are diagonally strict concave.

Appendices

Appendix A

Disruption Scenarios

AIRPORT	START TIME	DURATION	ORIGINAL CAPACITY	ADJUSTED CAPACITY
SCENARIO 1				
DCA	20:30	270	21	16
DCA	1:00	60	24	18
DCA	2:00	60	35	26
DCA	3:00	15	48	36
SCENARIO 2				
SFO	16:00	120	30	23
SCENARIO 3				
EWR	20:30	30	30	23
EWR	21:00	180	25	19
EWR	0:00	120	30	23
EWR	2:00	60	35	26
EWR	3:00	120	48	36
SCENARIO 4				
JFK	19:00	120	40	30
JFK	21:00	300	35	26
JFK	2:00	120	40	30
SCENARIO 5				
LGA	18:00	180	36	27
LGA	21:00	300	30	23
LGA	2:00	120	40	30
SCENARIO 6				
EWR	16:00	300	38	29
EWR	21:00	420	30	23
SCENARIO 7				

Table A.1 – continued from previous page

AIRPORT	START TIME	DURATION	ORIGINAL CAPACITY	ADJUSTED CAPACITY
ORD	19:00	60	72	54
ORD	20:00	180	76	57
ORD	23:00	150	84	63
SCENARIO 8				
MDW	22:15	150	22	17
SCENARIO 9				
JFK	17:00	120	54	41
JFK	19:00	300	36	27
JFK	0:00	120	38	29
JFK	2:00	120	54	41
SCENARIO 10				
EWR	17:00	120	38	29
EWR	19:00	180	30	23
EWR	22:00	360	38	29
SCENARIO 11				
SFO	16:00	60	30	23
SCENARIO 12				
SFO	17:00	75	30	23
SCENARIO 13				
LGA	17:00	120	38	29
LGA	19:00	240	28	21
LGA	23:00	270	38	29
SCENARIO 14				
IAD	19:00	60	34	26
IAD	20:00	240	30	23
IAD	0:00	90	34	26
SCENARIO 15				
EWR	17:00	180	38	29
SCENARIO 16				
SFO	15:30	150	30	23
SFO	18:00	15	48	36
SCENARIO 17				
LGA	19:15	390	38	29

Table A.1 – continued from previous page

AIRPORT	START TIME	DURATION	ORIGINAL CAPACITY	ADJUSTED CAPACITY
SCENARIO 18				
JFK	16:00	180	30	23
JFK	19:00	120	34	26
JFK	21:00	120	36	27
JFK	23:00	60	40	30
SCENARIO 19				
SFO	15:30	210	30	23
SFO	19:00	60	45	34
SCENARIO 20				
LGA	14:00	60	21	16
LGA	15:00	120	30	23
LGA	17:00	240	35	26
LGA	21:00	240	40	30
SCENARIO 21				
PHL	20:45	75	45	34
PHL	22:00	105	52	39
SCENARIO 22				
LGA	18:15	90	40	30
LGA	19:45	60	34	26
LGA	20:45	195	30	23
LGA	0:00	120	34	26
LGA	2:00	45	40	30
SCENARIO 23				
EWR	16:30	330	36	27
EWR	22:00	105	42	32
SCENARIO 24				
SFO	15:00	135	30	23
SCENARIO 25				
ORD	14:00	180	74	56
ORD	17:00	90	80	60
SCENARIO 26				
LGA	20:45	210	32	24
SCENARIO 27				

Table A.1 – continued from previous page

AIRPORT	START TIME	DURATION	ORIGINAL CAPACITY	ADJUSTED CAPACITY
EWR	18:00	60	38	29
EWR	19:00	120	40	30
EWR	21:00	120	34	26
EWR	23:00	60	36	27
		SCENARIO 28		
JFK	18:00	240	43	32
JFK	22:00	240	36	27
JFK	2:00	60	43	32
		SCENARIO 29		
PHL	0:15	135	36	27
		SCENARIO 30		
ATL	18:00	300	96	72
		SCENARIO 31		
LGA	21:30	210	34	26
LGA	1:00	60	15	11
LGA	2:00	60	25	19
LGA	3:00	30	34	26
		SCENARIO 32		
EWR	16:00	180	38	29
EWR	19:00	300	44	33
EWR	0:00	120	34	26
EWR	2:00	90	44	33
		SCENARIO 33		
JFK	21:00	60	40	30
JFK	22:00	240	35	26
JFK	2:00	30	44	33
		SCENARIO 34		
SFO	2:00	300	30	23
		SCENARIO 35		
IAD	14:30	165	40	30
		SCENARIO 36		
JFK	20:30	30	40	30
JFK	21:00	240	30	23

Table A.1 – continued from previous page

AIRPORT	START TIME	DURATION	ORIGINAL CAPACITY	ADJUSTED CAPACITY
JFK	1:00	60	35	26
JFK	2:00	60	40	30
JFK	3:00	45	45	34
SCENARIO 37				
PHL	20:00	60	35	26
PHL	21:00	240	30	23
PHL	1:00	60	35	26
PHL	2:00	60	40	30
PHL	3:00	60	45	34
SCENARIO 38				
IAD	20:30	270	21	16
IAD	1:00	60	40	30
IAD	2:00	60	50	38
SCENARIO 39				
ATL	17:00	120	104	78
ATL	19:00	30	102	77
ATL	19:30	210	94	71
ATL	23:00	60	96	72
ATL	0:00	120	104	78
SCENARIO 40				
LGA	19:45	15	40	30
LGA	20:00	60	30	23
LGA	21:00	180	25	19
LGA	0:00	60	30	23
LGA	1:00	105	35	26

Bibliography

[1] Douglas Fearing and Ian Kash. Managing air traffic disruptions through strategic prioritization.

[2] Bureau of Transportation Statistics (BTS). Major u.s. air carrier delays, cancellations, and diversions, 2014. URL `http://www.rita.dot.gov/bts/data_and_statistics/databases`.

[3] Federal Aviation Administration (FAA). The economic impact of civil aviation on the u.s. economy. Technical report, U.S. Department of Transportation, 2014.

[4] Michael Ball, Cynthia Barnhart, Martin Dresner, Mark Hansen, Kevin Neels, Amedeo Odoni, Everett Peterson, Lance Sherry, Antonio A Trani, and Bo Zou. Total delay impact study: a comprehensive assessment of the costs and impacts of flight delay in the united states. 2010.

[5] Amedeo R Odoni. The flow management problem in air traffic control. In *Flow control of congested networks*, pages 269–288. Springer, 1987.

[6] Jennifer Gentry, Kent Duffy, and William J. Swedish. Airport capacity profiles. Technical report, MITRE Corporation Center for Advanced Aviation System Development, 2014.

[7] Cynthia Barnhart, Dimitris Bertsimas, Constantine Caramanis, and Douglas Fearing. Equitable and efficient coordination in traffic flow management. *Transportation Science*, 46(2):262–280, 2012.

[8] Kan Chang, Ken Howard, Rick Oiesen, Lara Shisler, Midori Tanino, and Michael C Wambsganss. Enhancements to the faa ground-delay program under collaborative decision making. *Interfaces*, 31(1):57–76, 2001.

[9] Thomas WM Vossen and Michael O Ball. Slot trading opportunities in collaborative ground delay programs. *Transportation Science*, 40(1):29–43, 2006.

[10] K. Hoffman, F. Berardino, R. Golaszewski, L. Sherry, K. Aqbal, G. Hunter, D. Escala, and H. Gao. Results of a congestion management study for day-of-flight. Presentation at INFORMS TSL Workshop on Congestion Management of Transportation Systems on the Ground and in the Air, Asilomar, CA, 2011.

[11] Michael O Ball, Robert Hoffman, and Avijit Mukherjee. Ground delay program planning under uncertainty based on the ration-by-distance principle. *Transportation Science*, 44(1):1–14, 2010.

[12] Bengi Manley. Minimizing the pain in air transportation: analysis of performance and equity in ground delay programs. 2008.

[13] Stephen J Rassenti, Vernon L Smith, and Robert L Bulfin. A combinatorial auction mechanism for airport time slot allocation. *The Bell Journal of Economics*, pages 402–417, 1982.

[14] Eric John Cholankeril. *Evaluation of an auction mechanism for allocating airport arrival slots*. PhD thesis, Massachusetts Institute of Technology, 2003.

[15] Pavithra Harsha. *Mitigating airport congestion: market mechanisms and airline response models*. PhD thesis, Massachusetts Institute of Technology, 2009.

[16] Charles R Plott, David M Grether, and R Mark Isaac. *The allocation of scarce resources: Experimental Economics and the problem of allocating airport slots*. Westview Press, 1989.

[17] Michael Ball, George Donohue, and Karla Hoffman. Auctions for the safe, efficient, and equitable allocation of airspace system resources. *Combinatorial auctions*, 1, 2006.

[18] Jan K Brueckner. Airport congestion when carriers have market power. *American Economic Review*, pages 1357–1375, 2002.

[19] Jan K Brueckner. Internalization of airport congestion: A network analysis. *International Journal of Industrial Organization*, 23(7):599–614, 2005.

[20] J Ben Rosen. Existence and uniqueness of equilibrium points for concave n-person games. *Econometrica: Journal of the Econometric Society*, pages 520–534, 1965.

[21] Roger Beatty, Rose Hsu, Lee Berry, and James Rome. Preliminary evaluation of flight delay propagation through an airline schedule. *Air Traffic Control Quarterly*, 7(4):259–270, 1999.

[22] Douglas Fearing and Cynthia Barnhart. Evaluating air traffic flow management in a collaborative decision-making environment. *Transportation Research Record: Journal of the Transportation Research Board*, 2206(1):10–18, 2011.

[23] Gerard Debreu. A social equilibrium existence theorem. *Proceedings of the National Academy of Sciences of the United States of America*, 38(10):886, 1952.

[24] Irving L Glicksberg. A further generalization of the kakutani fixed point theorem, with application to nash equilibrium points. *Proceedings of the American Mathematical Society*, 3(1):170–174, 1952.

[25] Ky Fan. Fixed-point and minimax theorems in locally convex topological linear spaces. *Proceedings of the National Academy of Sciences of the United States of America*, 38(2):121, 1952.

[26] Metron Aviation. Flight schedule monitor, 2015. URL `http://www.metronaviation.com/solutions/traffic-flow-management/systems/fsm.html`.

[27] Thomas Vossen and Michael Ball. Optimization and mediated bartering models for ground delay programs. *Naval Research Logistics (NRL)*, 53(1):75–90, 2006.